ДИНАМИЧЕН АНАЛИЗ НА МАШИНИТЕ

1. Уравнение за движение при едномасов модел

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$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{q}_i}\right) - \frac{\partial}{\partial q_i}\left(E - \Pi\right) = Q_i$$

$E -$ *кинетична енергия на системата*
$\Pi -$ *потенциална енергия на системата*
$Q_i -$ *обобщена сила*
$q_i -$ *обобщена координата*
$\dot{q}_i -$ *обобщена скорост*

$$\frac{d}{dt}\left(\frac{\partial E}{\partial w}\right) - \frac{\partial E}{\partial j} = M_r$$

$$E = \frac{J_r w^2}{2}$$

$$M_r = M_D - M_C$$

$$\frac{d}{dt}\left(\frac{\partial E}{\partial w}\right) = \frac{d}{dt}\left(J_r w\right) = J_r \frac{dw}{dt} + w \frac{dJ_r}{dj} \frac{dj}{dt} = J_r \frac{dw}{dt} + w^2 \frac{dJ_r}{dj}$$

$$\frac{dE}{dj} = \frac{dJ_r}{dj} \frac{w^2}{2}$$

$$J_r \frac{dw}{dt} + \frac{w^2}{2} \frac{dJ_r}{dj} = M_r$$

2. Изменение на кинетичната енергия

$$\Delta\left(E + \Pi\right) = \Delta A$$

$$d\left(\frac{J_r w^2}{2}\right) = M_r dj$$

$$\int d\left(\frac{J_r w^2}{2}\right) = \int M_r dj$$

3. Алгоритъм за числено решаване

$$M_r\left(j, w, t\right); \quad J_r = J_r\left(j\right); \quad j_0; \quad w_0; \quad t_0.$$

$$w = w\left(j\right); \quad t = t\left(j\right) \Rightarrow w = w\left(t\right)$$

$$J_r \frac{dw}{dt} + \frac{w^2}{2}\frac{dJ_r}{dj} = M_r(j, w, t)$$

$$\frac{dw}{dt} = \frac{dw}{dj}\frac{dj}{dt} = w\frac{dw}{dj}$$

$$J_r w \frac{dw}{dj} + \frac{w^2}{2}\frac{dJ_r}{dj} = M_r(j, w, t)$$

$$J_r w dw + \frac{w^2 dJ_r}{2} = M_r(j, w, t) dj$$

$$J_r dw + \frac{w dJ_r}{2} = \frac{M_r(j, w, t)}{w} dj$$

$$2J_r dw + w dJ_r = \frac{2M_r(j, w, t)}{w} dj$$

$$dw = w_{i+1} - w_i; \quad dJ_r = J_{r\,i+1} - J_{r\,i}$$

$$w_{i+1} = \frac{M_r(j, w, t)}{J_{r\,i} w_i} dj + \frac{3J_{r\,i} - J_{r\,i+1}}{2J_{r\,i}} w_i$$

$$dt = \frac{dj}{w}; \; t_{i+1} - t_i = \frac{2\Delta j}{w_{i+1} + w_i}; \; t_{i+1} = t_i + \frac{2\Delta j}{w_{i+1} + w_i}$$

4. Частни случаи

$$M_r = M_r(j); \quad J_r = J_r(j)$$

$$\int_{j_0}^{j} M_r(j) = J_r(j)\frac{w^2}{2} - J_r(j_0)\frac{w_0^2}{2}$$

$$A(j) = \int_{j_0}^{j} M_r(j)$$

$$w = \sqrt{\frac{2}{J_r(j)}\left[J_r(j_0)\frac{w_0^2}{2} + A(j)\right]} \quad \Rightarrow w = w(j)$$

$$dt = \frac{dj}{w(j)}; \quad t_0 = 0, j_0 = 0.$$

$$t = \int_{j_0}^{j} \frac{dj}{\sqrt{\frac{2}{J_r(j)}\left[J_r(j_0)\frac{w_0^2}{2} + A(j)\right]}} \quad \Rightarrow t = t(j)$$

$$M_r = M_r(w); \quad J_r = J_0 = Const$$

$$M_r(w) = J_0 w \frac{dw}{dj}$$

$$M_r(w) = J_0 \frac{dw}{dt}; \quad t_0 = 0, j_0 = 0.$$

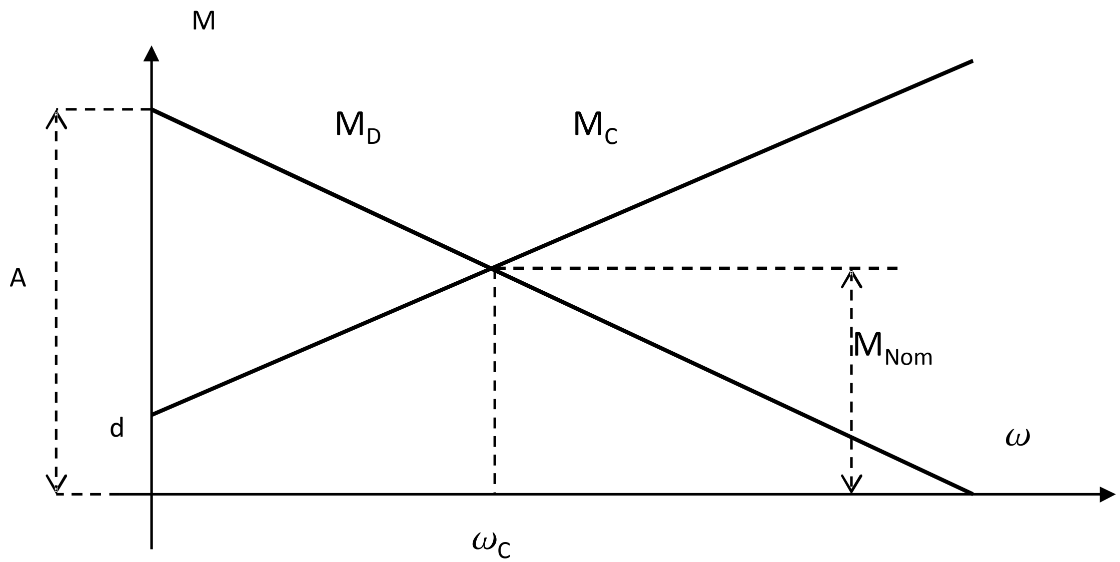
$$j = \int_{w_0}^{w} \frac{w dw}{M_r(w)} \quad \Rightarrow j = j(w)$$

$$t = \int_{w_0}^{w} \frac{dw}{M_r(w)} \quad \Rightarrow t = t(w)$$

$$\left(\frac{dw}{dt}\right)_{t=0} = e_0 = \frac{M_0}{J_0}$$

$$t_0 = \frac{w_C}{e_0} = \frac{w_C J_0}{M_0} \Rightarrow \text{временконстанта}$$

$$t_{пусково} \Rightarrow w_{t_{пусково}} = 0{,}95 w_C$$



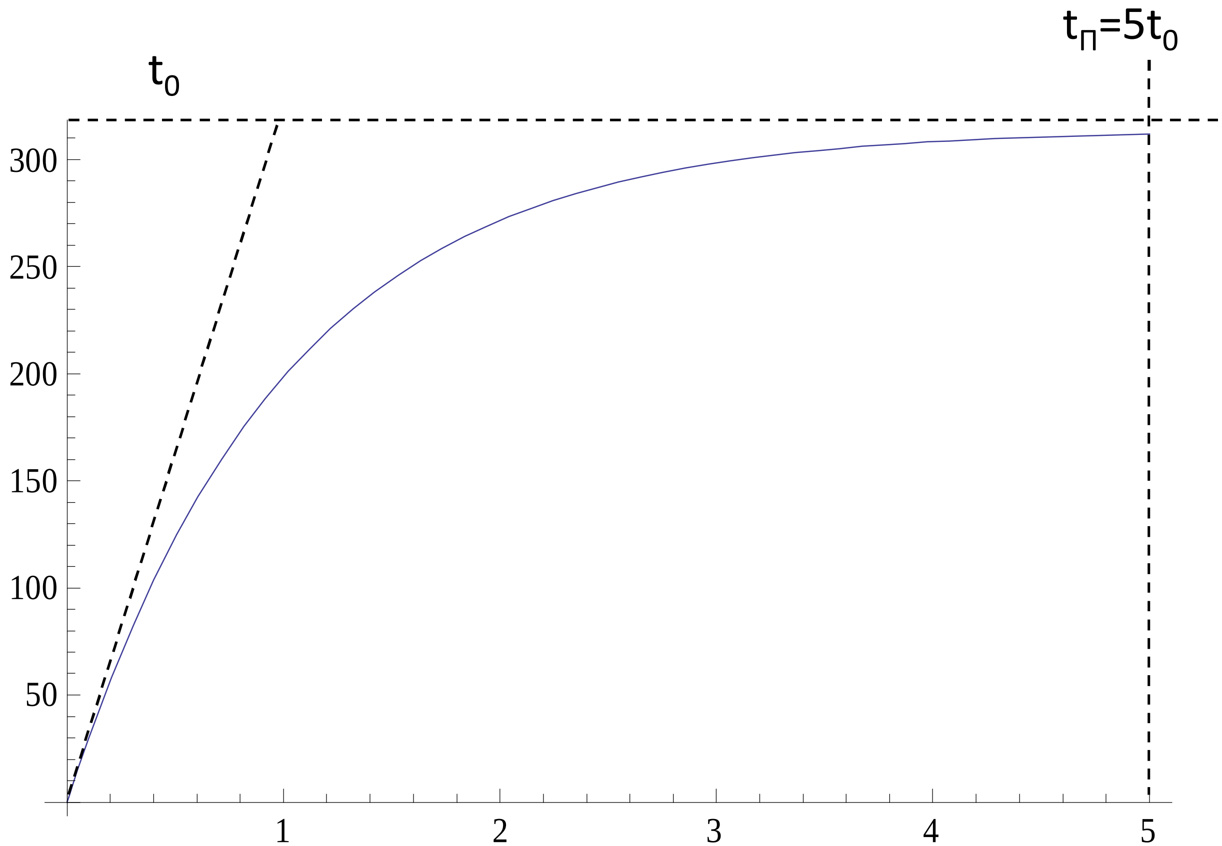
$$M_D = M_D(\omega) = A - k_D\omega;\quad M_C = M_C(\omega) = d + k_C\omega$$

$$M_r = M_r(\omega) = (A - d) - (k_D + k_C)\omega;\quad J_r = J_0 = Const$$

$$J_r\dot{\omega} = (A - d) - (k_D + k_C)\omega;\quad t_0 = 0, \varphi_0 = 0.$$

$$\frac{J_r}{(k_D + k_C)}\dot{\omega} + \omega = \frac{(A - d)}{(k_D + k_C)} = \omega_C \Rightarrow \tau_M\dot{\omega} + \omega = \omega_C;\qquad \tau_M = \frac{J_r}{(k_D + k_C)}$$

$$\omega = \omega_C\left(1 - e^{-\frac{t}{\tau_M}}\right).$$



t_0
$t_П=5t_0$
300
250
200
150
100
50
1
2
3
4
5

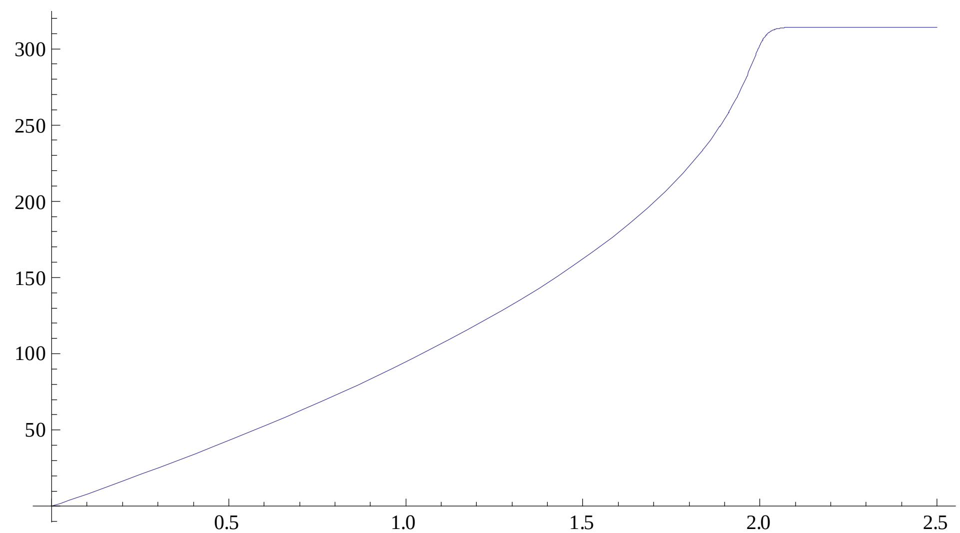
$$J_r \frac{dw}{dt} = M_r(w)$$

$$J_r dw = M_r(w) dt$$

$$J_r \left(w_{i+1} - w_i\right) = M_r(w)\left(t_{i+1} - t_i\right)$$

$$w_{i+1} = w_i + \frac{M_r(w)\Delta t}{J_r}$$

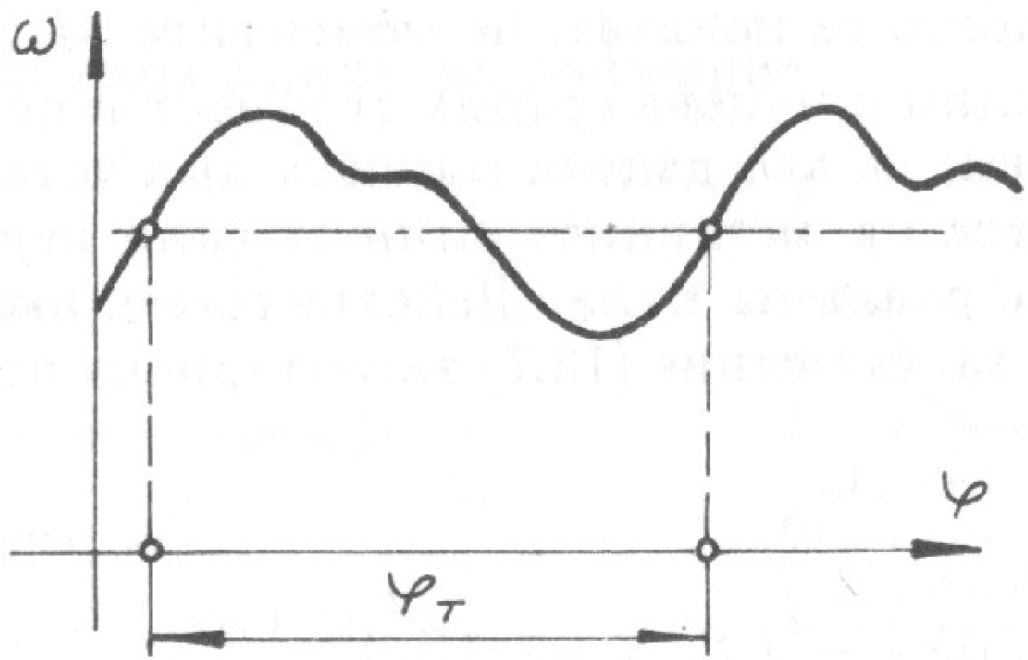
$$M_D(w) = \frac{2M_{Max}\left(w_S - w\right)}{\left(w_S - w\right)^2 + w_S^2 S_K^2}$$



Режими на движение

- Установен режим

$$\omega(t) = \omega(t + T), \qquad \omega(\varphi) = \omega(\varphi + \varphi_T).$$



- Условия за съществуване на установен режим при машини с една степен на свобода:

1. Приведения масов инерционен момент и приведения момент на активните сили да бъдат периодична функция на положението. Допустима е също масовите и силовите параметри да не зависят от положението.

$$J_r(\varphi) = J_r(\varphi + \varphi_J),$$

$$M_r(\varphi, \omega) = M_r(\varphi + \varphi_M, \omega)$$

2.Периодите на масите и силите да бъдат кратни.

$$p\varphi_J = q\varphi_M,$$

3. Приведения масов на активните сили да зависи от скоростта, като производната му спрямо нея да бъде отрицателна.

$$\frac{\partial M_r(\varphi,\omega)}{d\omega} < 0. \qquad \varphi_T = p\varphi_J = q\varphi_M.$$

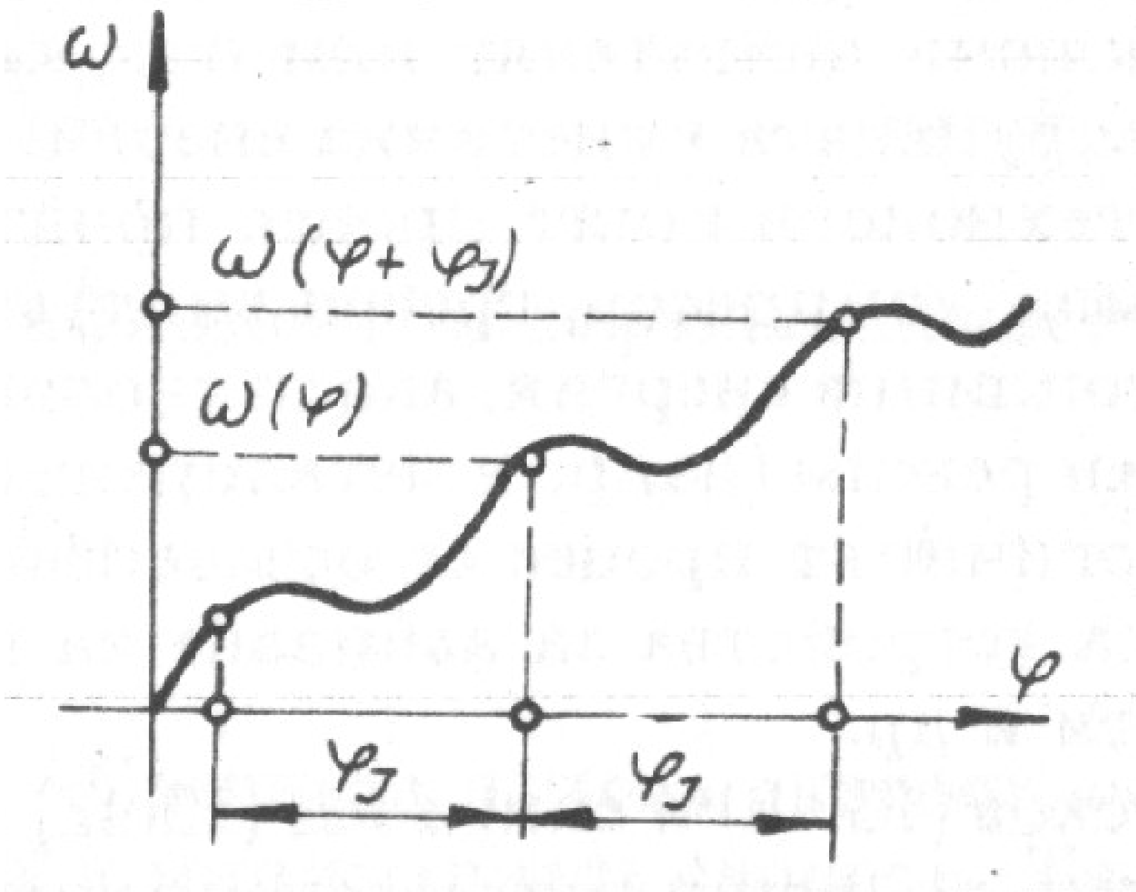
Ако тези три условия са спазени, машината има установен режим с ъглов период. Работата на активните сили за един период е равна на нула.

$$\int_{\varphi}^{\varphi+\varphi_T} M_r(\varphi,\omega)d\varphi = J_r(\varphi + p\varphi_J)\frac{\omega^2(\varphi+\varphi_T)}{2} - J_r(\varphi)\frac{\omega^2(\varphi)}{2} = 0,$$

$$\int_{0}^{\varphi_T} M_r(\varphi,\omega_e)d\varphi = 0,$$

- Пусков режим – кинетичната енергия расте.

$$\omega(\varphi + \varphi_J) > \omega(\varphi),$$

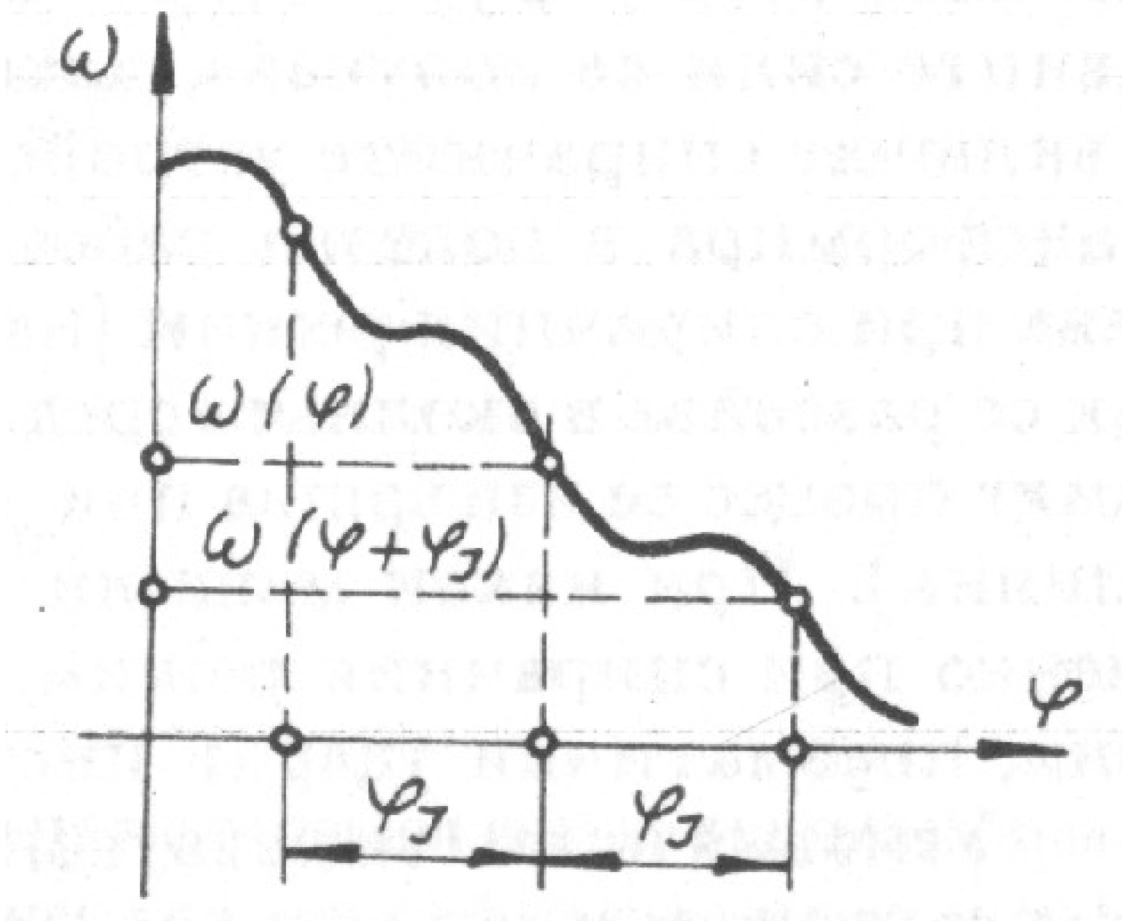


Работата на активните сили за един период е положителна.

$$\int_{\varphi}^{\varphi+\varphi_J} M_r(\varphi,\omega)d\varphi = J_r(\varphi+\varphi_J)\frac{\omega^2(\varphi+\varphi_T)}{2} - J_r\frac{\omega^2(\varphi)}{2} > 0,$$

- Спирачен режим – кинетичната енергия намалява.

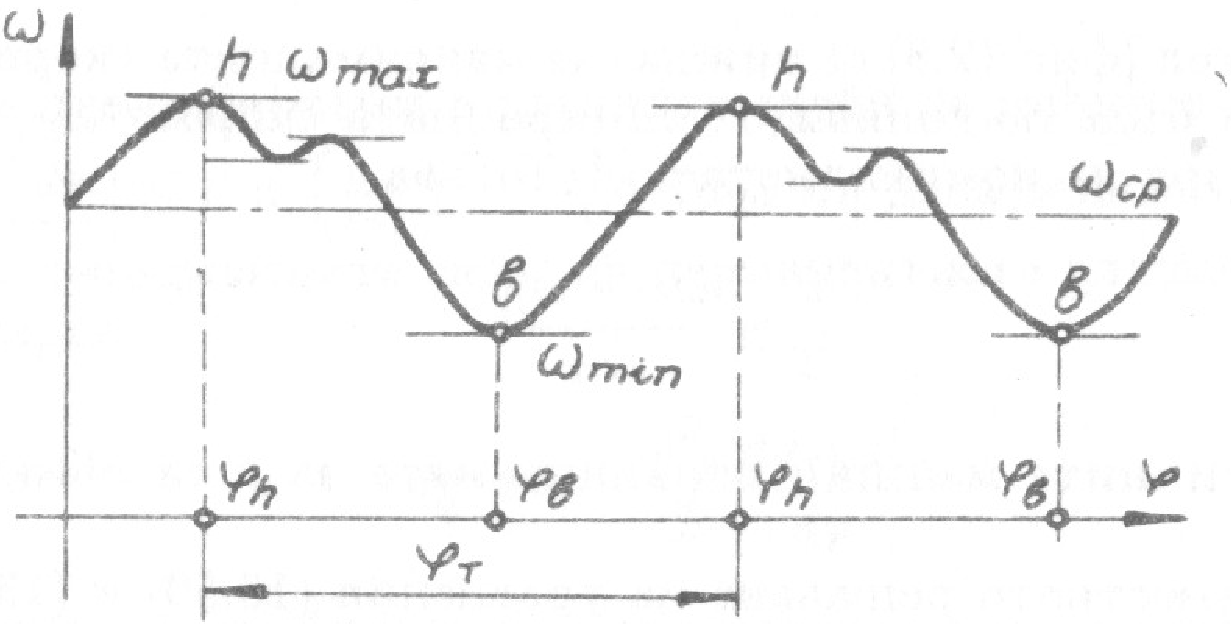
$$\omega(\varphi + \varphi_J) < \omega(\varphi),$$



Работата на активните сили за един период е отрицателна.

$$\int_{\varphi}^{\varphi+\varphi_J} M_r(\varphi,\omega)d\varphi = J_r(\varphi+\varphi_J)\frac{\omega(\varphi+\varphi_J)}{2} - J_r(\varphi)\frac{\omega^2(\varphi)}{2} < 0,$$

Качествен анализ на установеното движение



$$\omega_{cp} = \frac{\omega_{max} + \omega_{min}}{2} \qquad \delta = \frac{\omega_{max} - \omega_{min}}{\omega_{cp}}.$$

$$\varphi_{min} \Rightarrow \omega_{min} \;\; ; \varphi_{max} \Rightarrow \omega_{max}$$

$$\frac{J_{r(\varphi_{max})}\omega_{max}^2 - J_{r(\varphi_{min})}\omega_{min}^2}{2} = \int_{\varphi_{min}}^{\varphi_{max}} M_r(\varphi)d\varphi = \Delta A_{max}$$

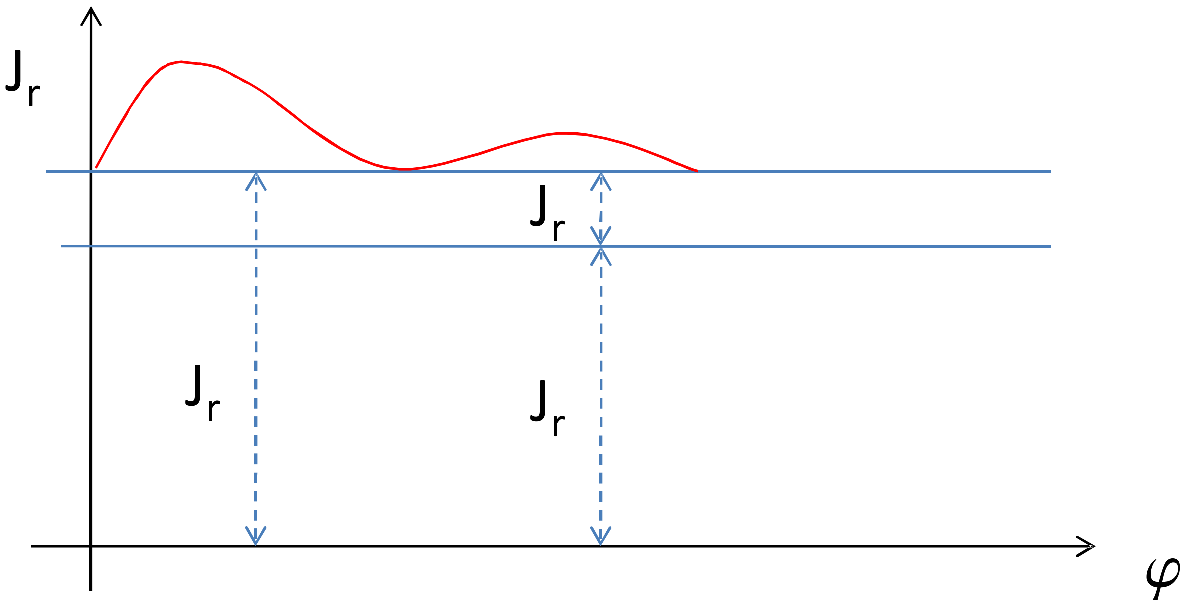
$$\Delta A_{max} = \int_{\varphi_{min}}^{\varphi_{max}} M_r(\varphi)d\varphi = \int_{\varphi_{min}}^{\varphi_{max}} (M_D - M_C)d\varphi$$

$$J_{r(\varphi_{max})} = J_{r(\varphi_{min})} = J$$

$$J(\omega_{max} - \omega_{min})\left(\frac{\omega_{max} + \omega_{min}}{2}\right) = \Delta A_{max}$$

$$\delta = \frac{\Delta A_{max}}{J\omega_{cp}^2}$$

Маховик



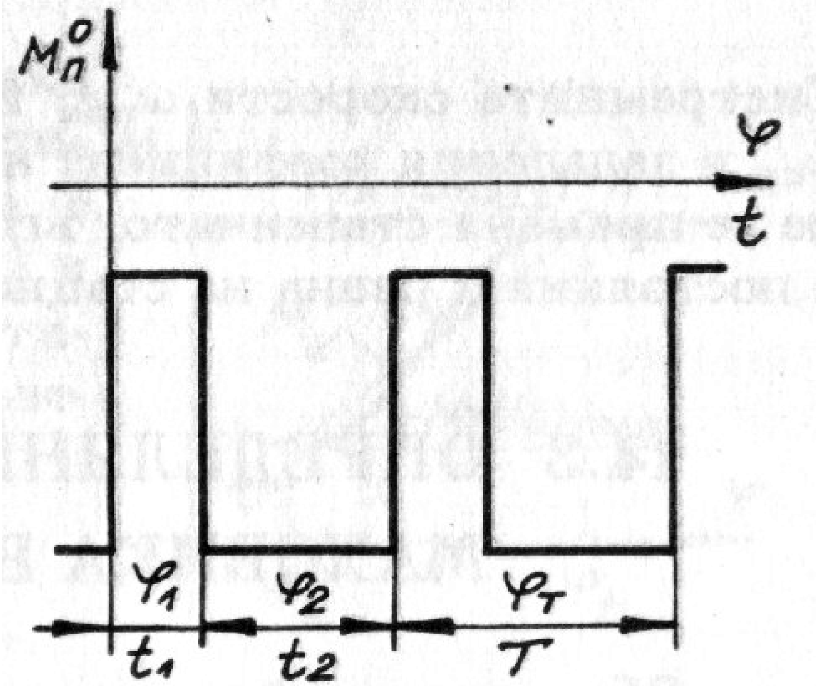
$$J_0 = J_C + J_M$$

$$J_C + J_M \approx \frac{\Delta A_{\max}}{dw_{cp}^2}; \quad J_M \approx \frac{\Delta A_{\max}}{dw_{cp}^2} - J_C$$

$$\left| \begin{array}{l} w_{\max} = w_{cp}\left(1 + \dfrac{d}{2}\right) \\ w_{\min} = w_{cp}\left(1 - \dfrac{d}{2}\right) \end{array} \right.$$

Частен случай

$$J_r = J_0 = \text{const}, \quad M_r = M_П^0 + M_K(\omega)$$



$$M_П^0 + M_K(\omega) = J_0 \omega \frac{d\omega}{d\varphi},$$

$$\int_0^{\varphi_1} d\varphi = J_0 \int_{\omega_{min}}^{\omega_{max}} \frac{\omega d\omega}{M_K(\omega) + M_П^0}.$$