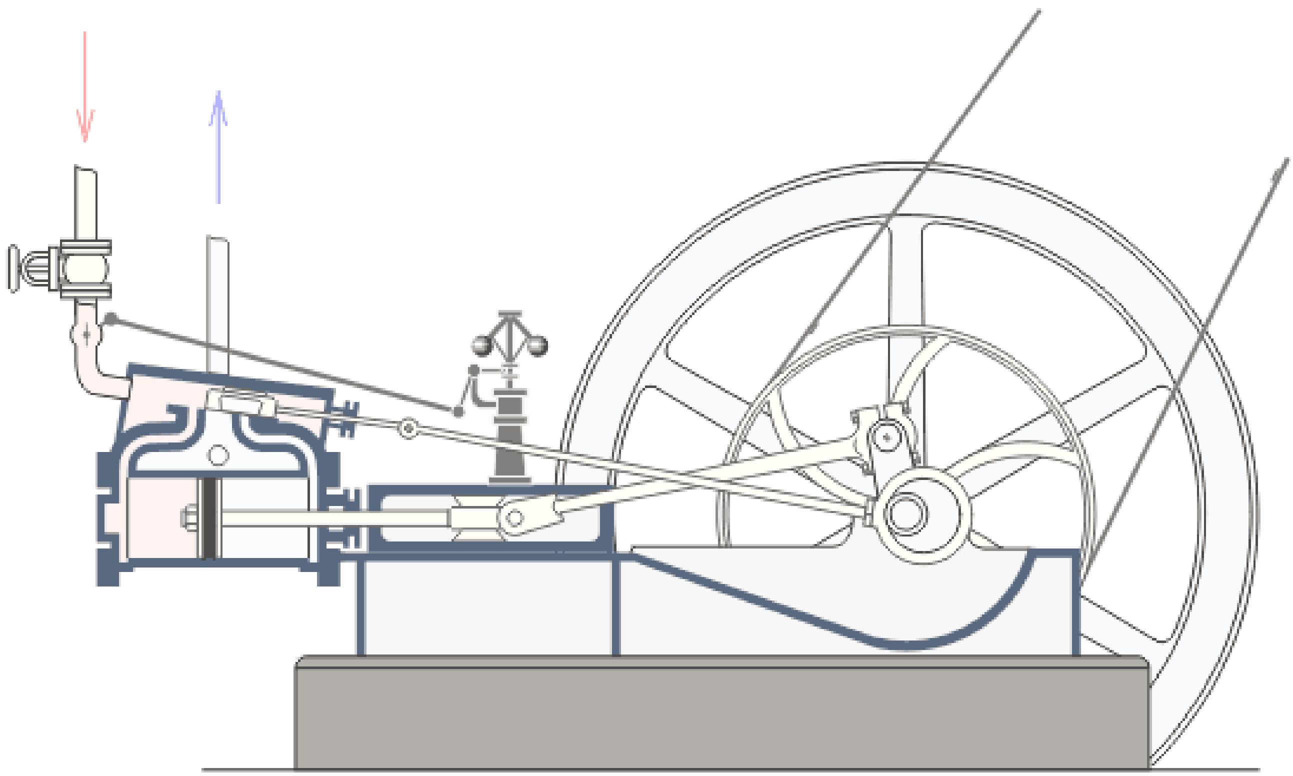
Аналитични методи за определяне на кинематичните параметри

1. Метод на непосредственото диференциране.
2. Метод на триъгълниците.
3. Графично диференциране.
4. Графично интегриране.

Парна машина

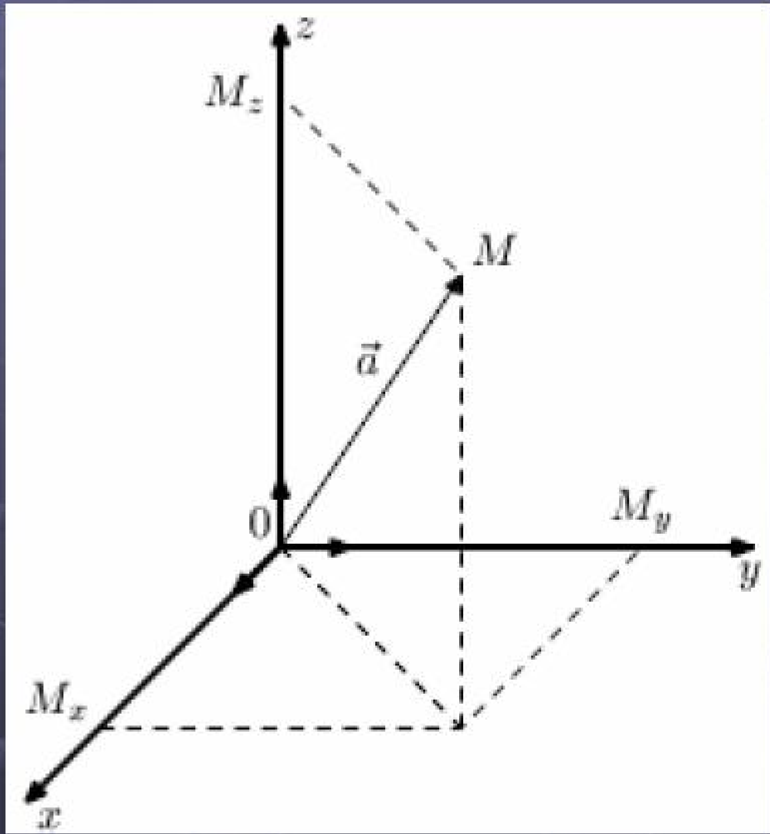


Координатен метод

Правоъгълна декартова координатна система

Закон за движение

$$x = x(t)$$
$$y = y(t)$$
$$z = z(t)$$

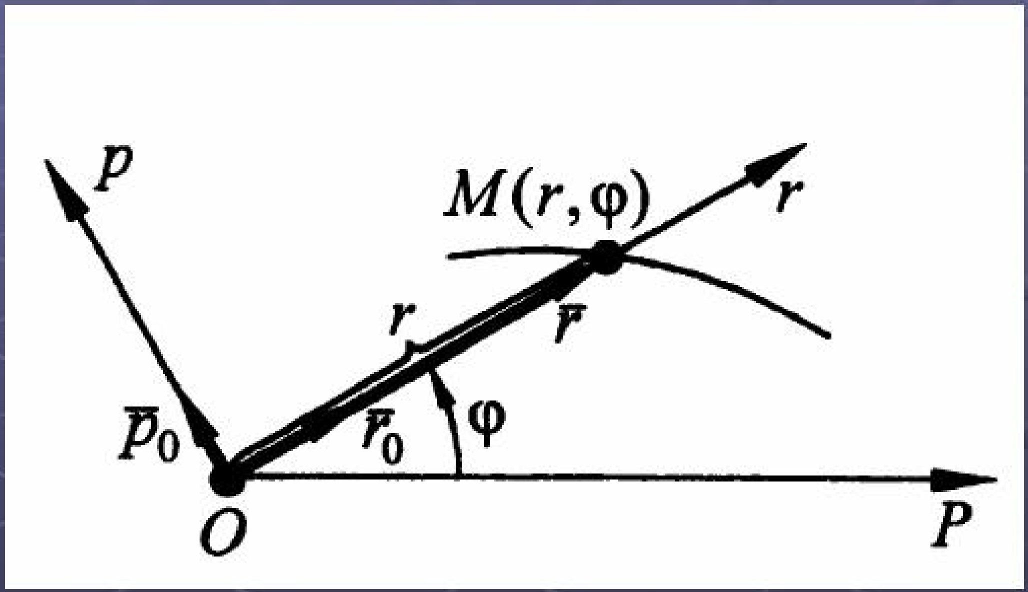


Траектория

$$x = x(t)$$
$$t = x^{-1}(x)$$
$$\Rightarrow \; y = y(t) = f_1(x)$$
$$\Rightarrow \; z = z(t) = f_2(x)$$

Координатен метод

Полярна координатна система



P -полярна ос

r -полярен радиус

φ -полярен ъгъл

Единични вектори

$$\vec{r}_0 = \frac{\partial \vec{r}}{\partial r}; \quad \vec{p}_0 = \frac{\partial \vec{r}}{\partial \varphi}$$

Координатни линии

$r = const$

$\varphi = const$

Закон за движение

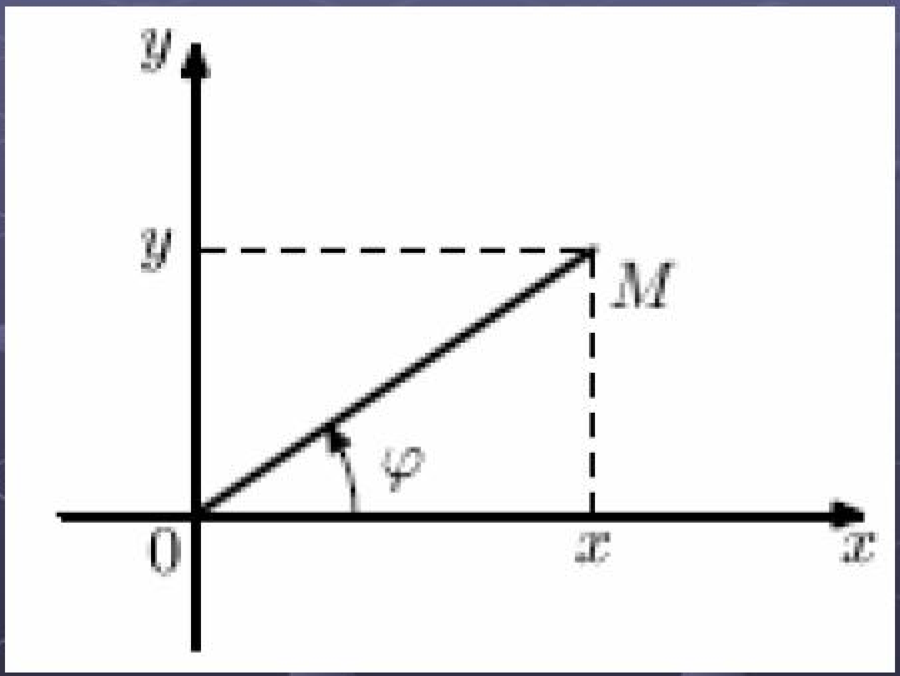
$r = r(t)$

$\varphi = \varphi(t)$

Координатен метод

Полярна координатна система

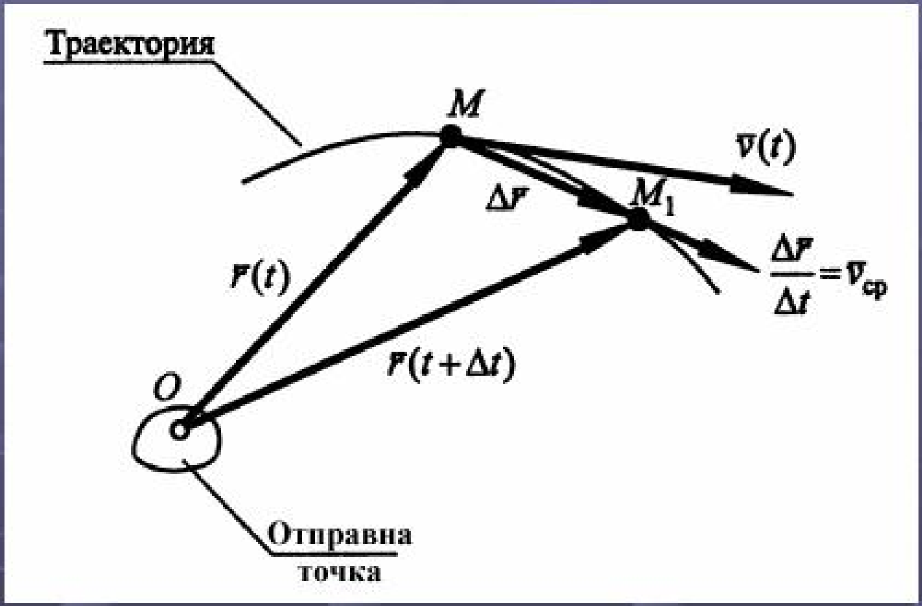
Връзка с декартовата координатна система



$$x = r.\cos\varphi$$
$$y = r.\sin\varphi$$

Скорост и ускорение на точка

Скорост. Векторно представяне



$$\Delta t = t' - t\ ;\ \ \Delta\vec{r} = \vec{r}(t') - \vec{r}(t)$$

$$\vec{v}_{ср} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{v}(t) = \lim_{\Delta t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \dot{\vec{r}}$$

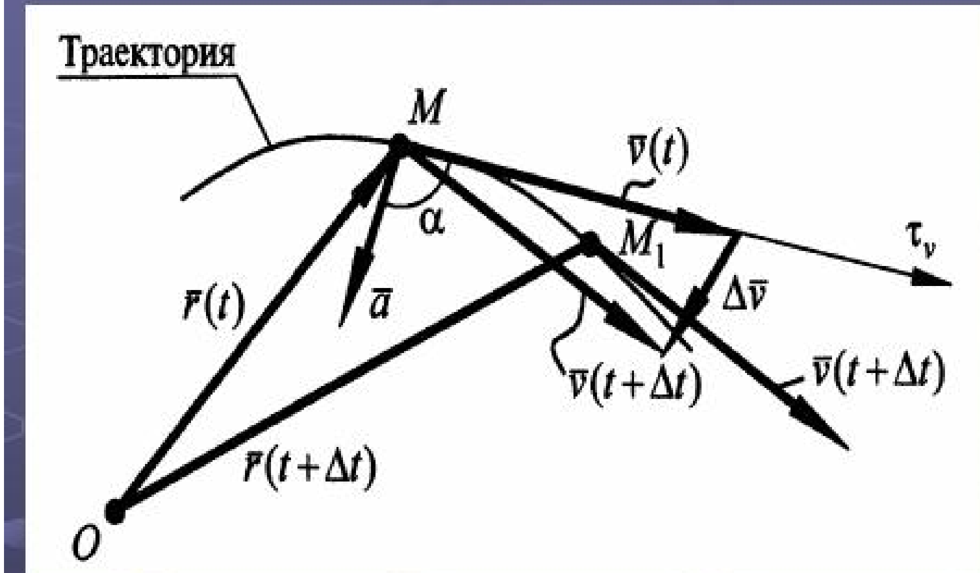
Изминат път:

$$L = \lim_{|\Delta\vec{r}_k|\to 0}\sum_k |\Delta\vec{r}_k| = \lim_{|\Delta t_k|\to 0}\sum_k \frac{|\Delta\vec{r}_k|}{\Delta t_k}\Delta t_k = \int_{t_1}^{t_2} v(t)\,dt$$

Скоростта е векторна физическа величина равна на първата производна по времето от радиус вектора на точката. Винаги е насочена по допирателната към траекторията.

Скорост и ускорение на точка

Ускорение. Векторно представяне



$$\Delta t = t' - t\ ;\quad \Delta\vec{v} = \vec{v}(t') - \vec{v}(t)$$

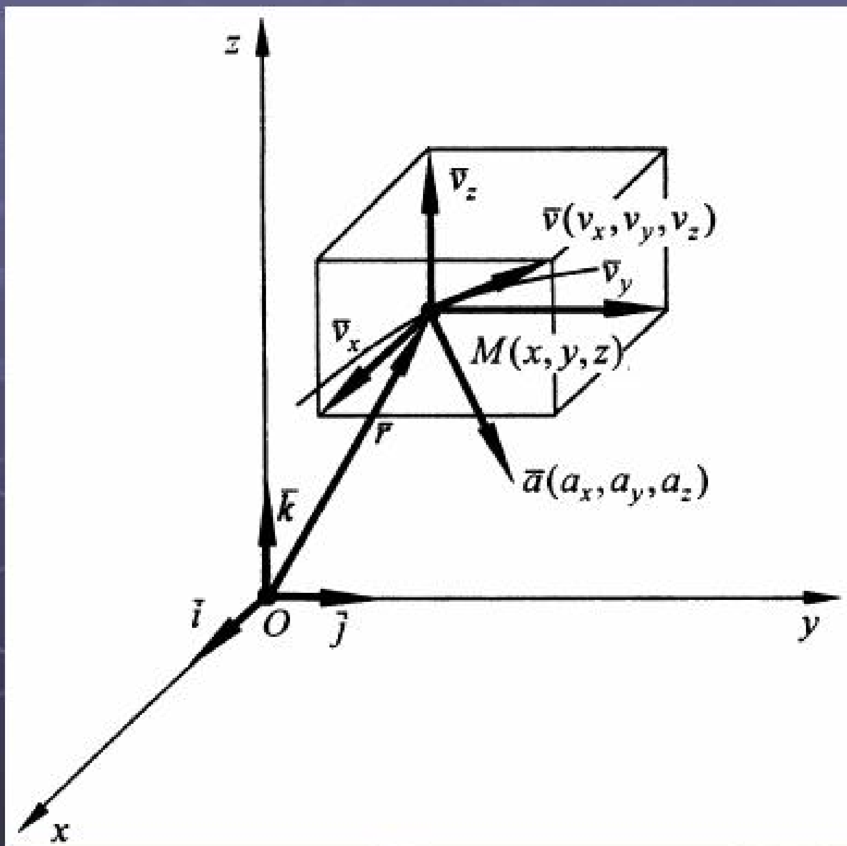
$$\vec{a}_{cp} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{a}(t) = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \dot{\vec{v}} = \ddot{\vec{r}}$$

Ускорението е векторна физическа величина равна на първата производна по времето от скоростта на точката. Винаги е насочена към вдлъбнатата страна на траекторията.

Скорост и ускорение на точка

Скорост в декартова координатна система



$$\vec{r}(t) = x(t).\vec{i} + y(t).\vec{j} + z(t).\vec{k}$$

$$\vec{v} = \dot{\vec{r}} = \dot{x}(t).\vec{i} + \dot{y}(t).\vec{j} + \dot{z}(t).\vec{k}$$

$$v_x = \vec{v}.\vec{i} = \dot{x}(t)$$

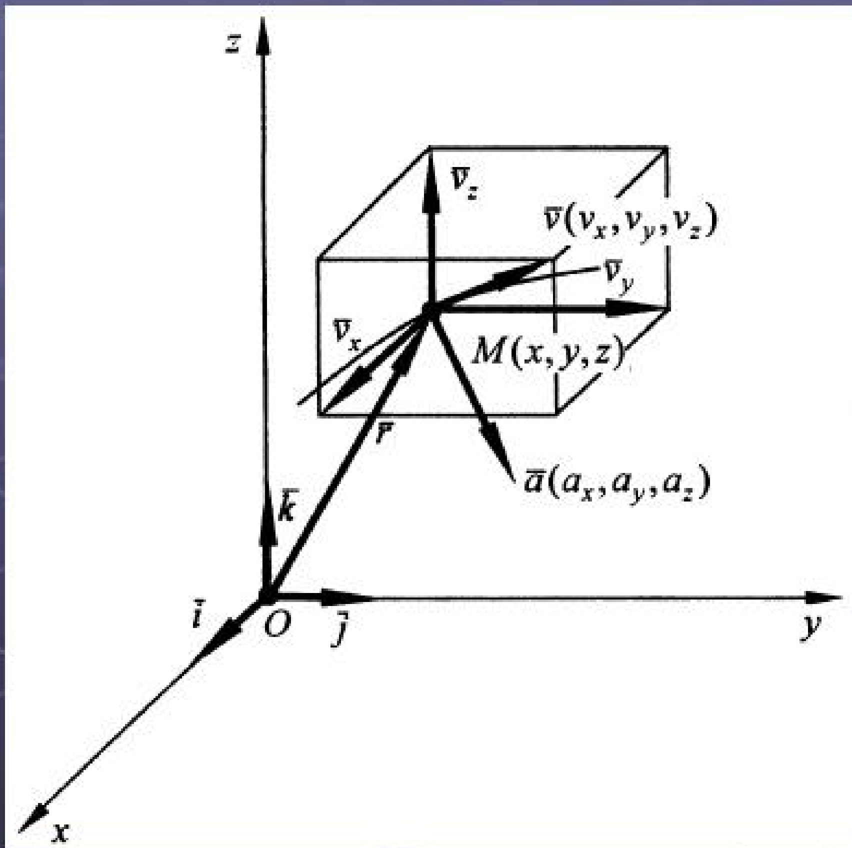
$$v_y = \vec{v}.\vec{j} = \dot{y}(t)$$

$$v_z = \vec{v}.\vec{k} = \dot{z}(t)$$

$$|\vec{v}| = \sqrt{\dot{x}(t)^2 + \dot{y}(t)^2 + \dot{z}(t)^2}$$

Скорост и ускорение на точка

Ускорение в декартова координатна система



$$\vec{v} = \dot{x}(t).\vec{i} + \dot{y}(t).\vec{j} + \dot{z}(t).\vec{k}$$

$$\vec{a} = \dot{\vec{v}} = \ddot{x}(t).\vec{i} + \ddot{y}(t).\vec{j} + \ddot{z}(t).\vec{k}$$

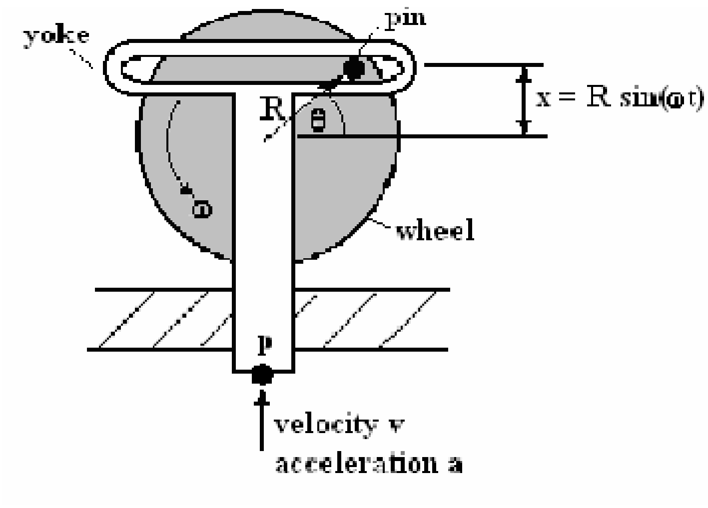
$$a_x = \vec{a}.\vec{i} = \ddot{x}(t) = \dot{v}_x$$

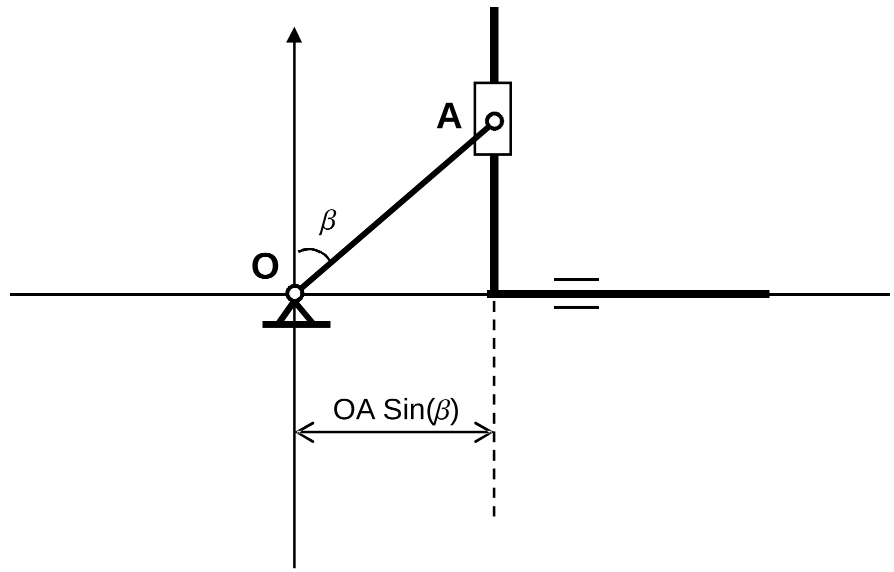
$$a_y = \vec{a}.\vec{j} = \ddot{y}(t) = \dot{v}_y$$

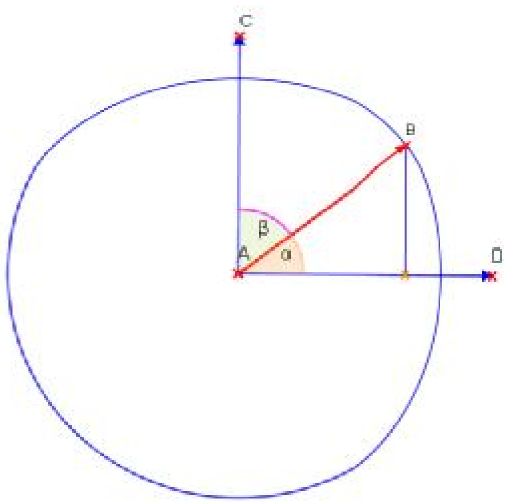
$$a_z = \vec{a}.\vec{k} = \ddot{z}(t) = \dot{v}_z$$

$$|\vec{a}| = \sqrt{\ddot{x}(t)^2 + \ddot{y}(t)^2 + \ddot{z}(t)^2}$$

Метод на непосредственото диференциране.







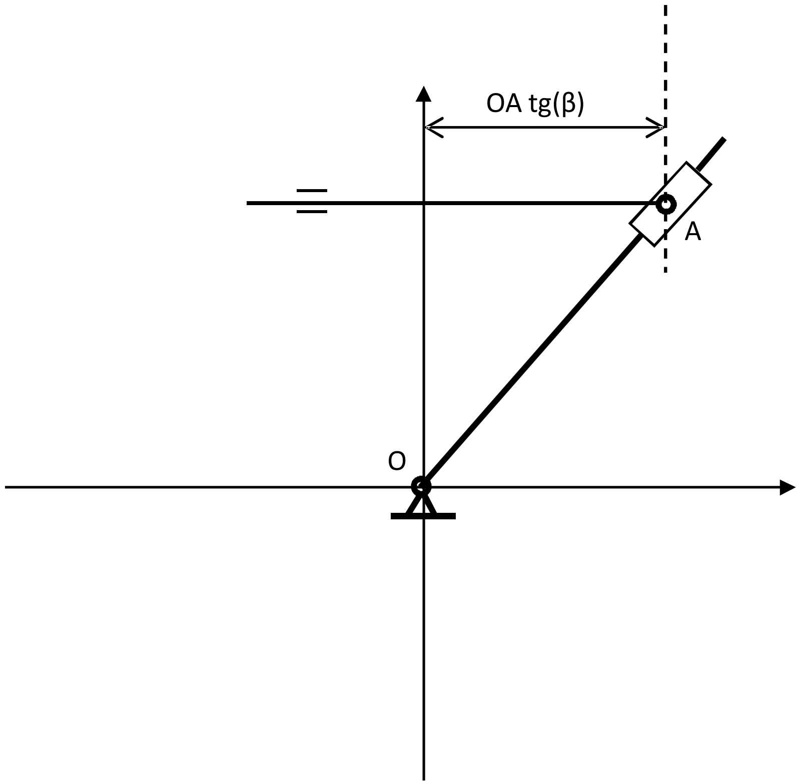
$$\mathrm{x} = \mathrm{OA\,Sin}(b)$$

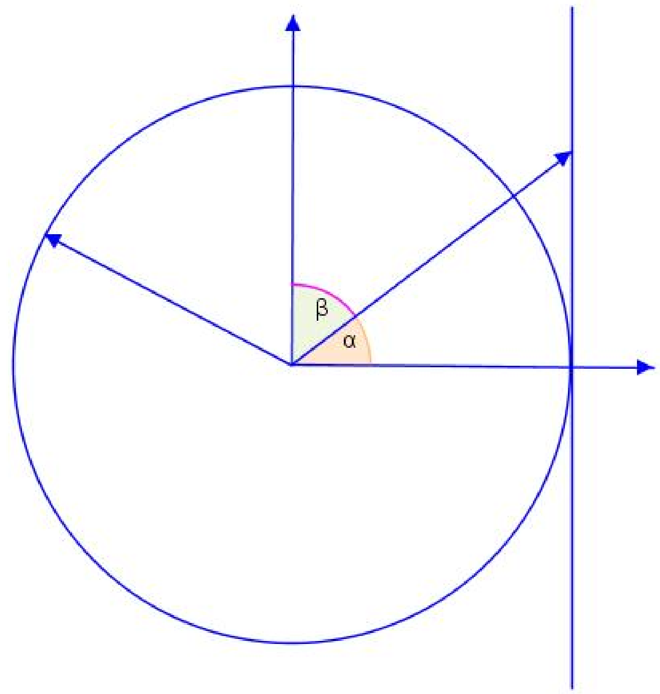
$$\dot{x} = \frac{dx}{dt} = \frac{dx}{db}\frac{\mathrm{d}b}{\mathrm{dt}} = \mathrm{OACos}(b)\frac{\mathrm{d}b}{\mathrm{dt}} =$$

$$= \mathrm{OACos}(b)\; w = \mathrm{OACos}(w\,t)\; w$$

$$\ddot{x} = \frac{d^2x}{dt^2} = \frac{d\dot{x}}{db}\frac{\mathrm{d}b}{\mathrm{dt}} = \frac{d^2x}{db^2}\left(\frac{\mathrm{d}b}{\mathrm{dt}}\right)^2 + \frac{dx}{db}\frac{d^2b}{dt^2} =$$

$$= -\mathrm{OA\,Sin}(b)\left(\frac{\mathrm{d}b}{\mathrm{dt}}\right)^2 + \mathrm{OACos}(b)\,\frac{d^2b}{dt^2} = -\mathrm{OA\,Sin}(b)\; w^2 + \mathrm{OACos}(b)\; e$$





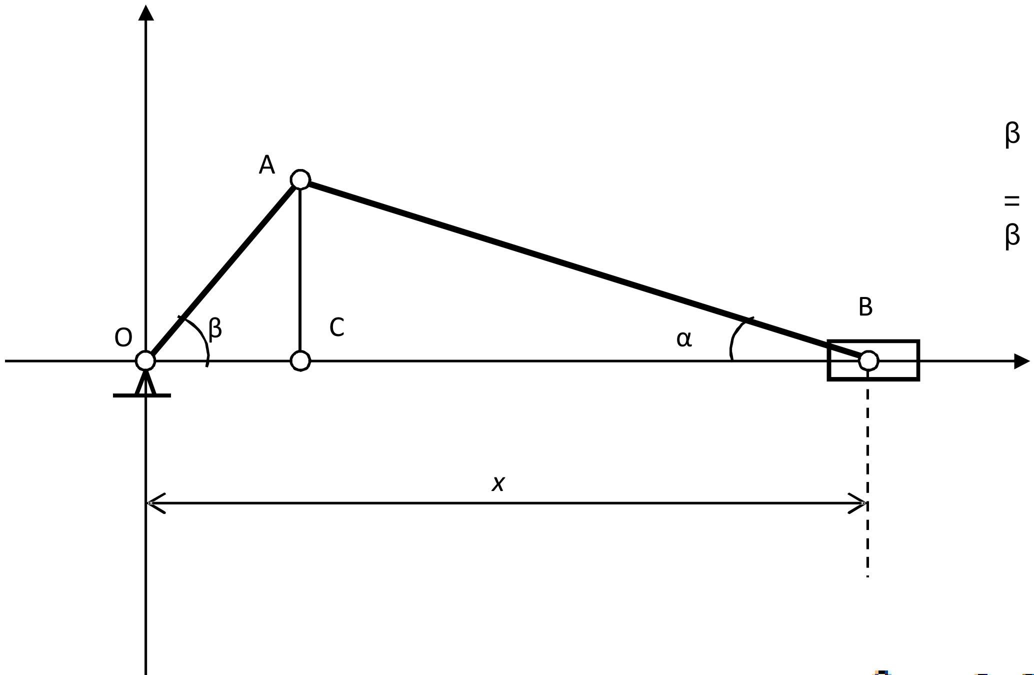
$$x = \mathrm{OA}\,\mathrm{Tg}(b)\ ;\quad b = w\ t$$

$$\dot{x} = \frac{dx}{dt} = \frac{dx}{db}\frac{\mathrm{d}b}{\mathrm{dt}} = \mathrm{OA}\frac{1}{\mathrm{Cos}^2(b)}\frac{\mathrm{d}b}{\mathrm{dt}} =$$

$$= \mathrm{OACos}(b)\ w$$

$$\ddot{x} = \frac{d^2x}{dt^2} = \frac{d\dot{x}}{db}\left(\frac{\mathrm{d}b}{\mathrm{dt}}\right) = \frac{d^2x}{db^2}\left(\frac{\mathrm{d}b}{\mathrm{dt}}\right)^2 + \frac{dx}{db}\frac{d^2b}{dt^2} =$$

$$= -\mathrm{OA}\frac{2\mathrm{Tg}(b)}{\mathrm{Cos}^2(b)}\left(\frac{\mathrm{d}b}{\mathrm{dt}}\right)^2 + \mathrm{OA}\frac{1}{\mathrm{Cos}^2(b)}\frac{d^2b}{dt^2} = -\mathrm{OA}\frac{2\mathrm{Tg}(b)}{\mathrm{Cos}^2(b)}w^2 + \mathrm{OA}\frac{1}{\mathrm{Cos}^2(b)}\ e$$



$$x = OC + CB = OA\ Cos(b) + AB\ Cos(a)$$

β = β

$$\frac{OA}{Sin(a)} = \frac{AB}{Sin(b)} \Rightarrow a = ArcSin\left[\frac{OA}{AB} Sin(b)\right]$$

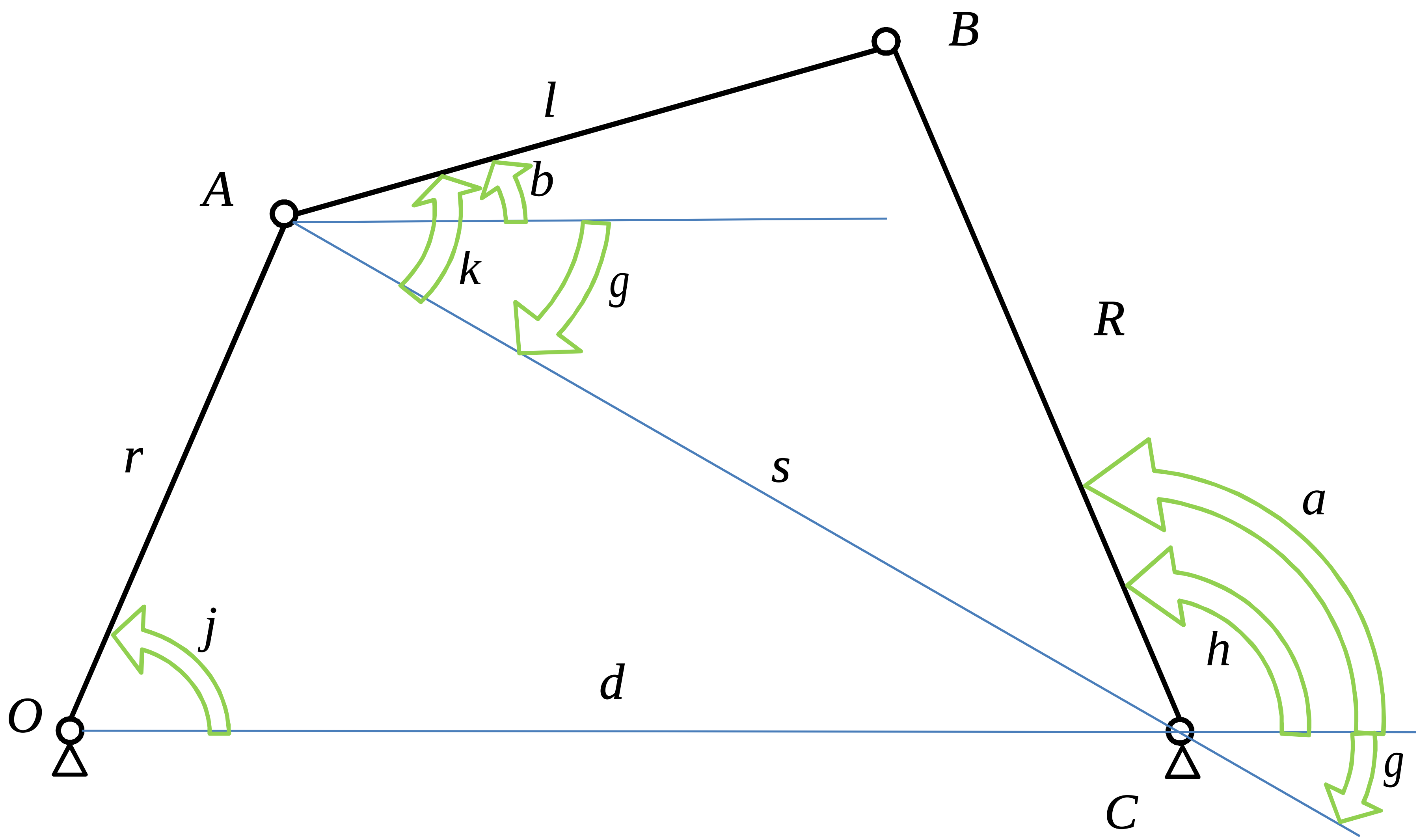
$$x = OACos[\beta] + AB\sqrt{1 - \frac{OA^2 Sin[\beta]^2}{AB^2}}$$

$$\dot{x} = \frac{dx}{dt} = \frac{dx}{d\beta}\frac{d\beta}{dt} = -OASin[\beta] - \frac{OA^2 Cos[\beta] Sin[\beta]}{AB\sqrt{1 - \frac{OA^2 Sin[\beta]^2}{AB^2}}}$$

$$\ddot{x} = \frac{d^2x}{dt^2} = \frac{d\dot{x}}{d\beta}\frac{d\beta}{dt} = \frac{d^2x}{d\beta^2}\left(\frac{d\beta}{dt}\right)^2 + \frac{dx}{d\beta}\frac{d^2\beta}{dt^2}$$

$$= \quad OACos[\beta] \;|\; AB\left(\frac{OA^4 Cos[\beta]^2 Sin[\beta]^2}{AB^4\left(1 - \frac{OA^2 Sin[\beta]^2}{AB^2}\right)^{\frac{3}{2}}} \quad \frac{OA^2 Cos[\beta]^2}{AB^2\sqrt{1 - \frac{OA^2 Sin[\beta]^2}{AB^2}}} \;|\; \frac{OA^2 Sin[\beta]^2}{AB^2\sqrt{1 - \frac{OA^2 Sin[\beta]^2}{AB^2}}}\right)$$

Метод на триъгълниците.



B
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R
r
s
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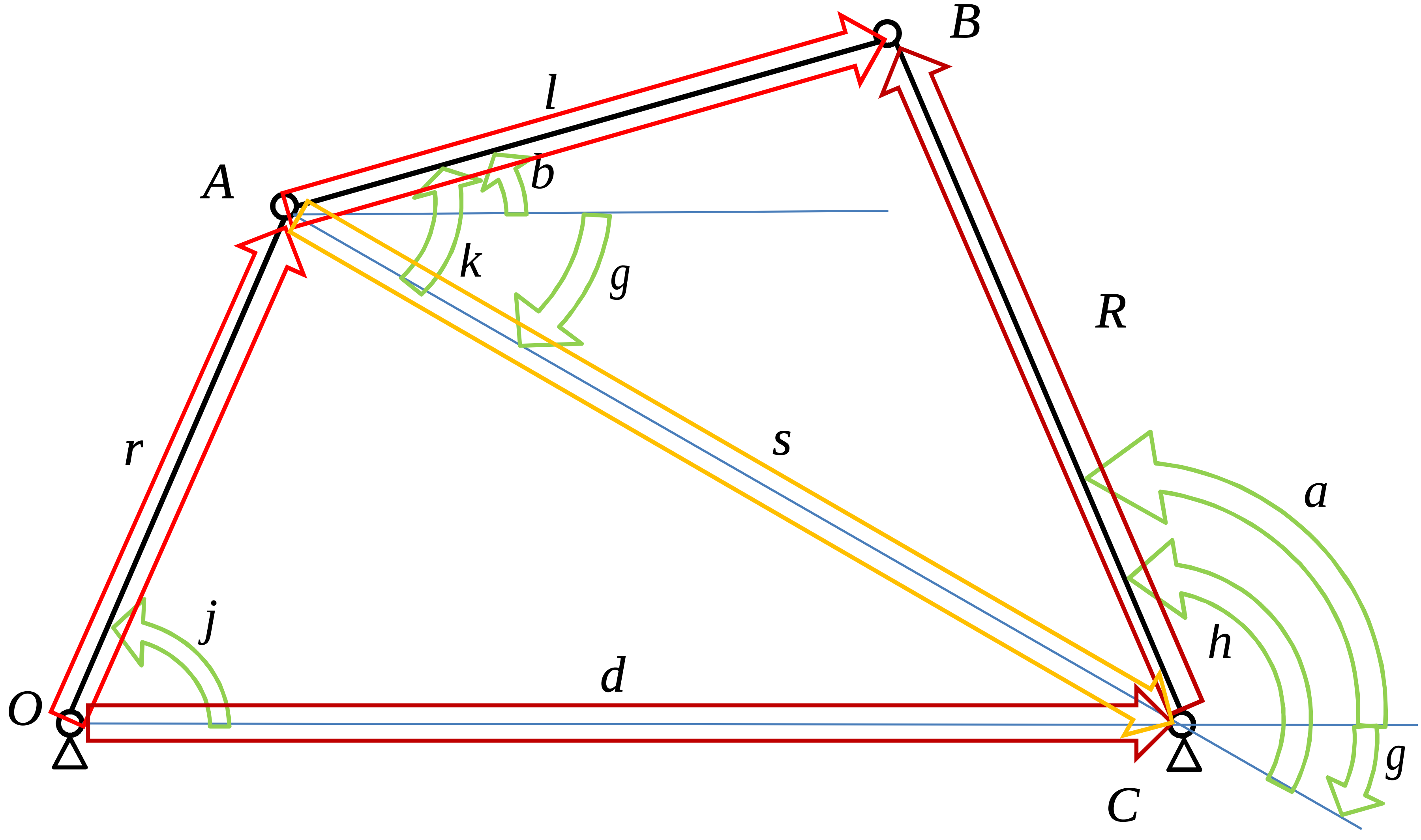
j
h
O
d
g
C


B
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R
s
r
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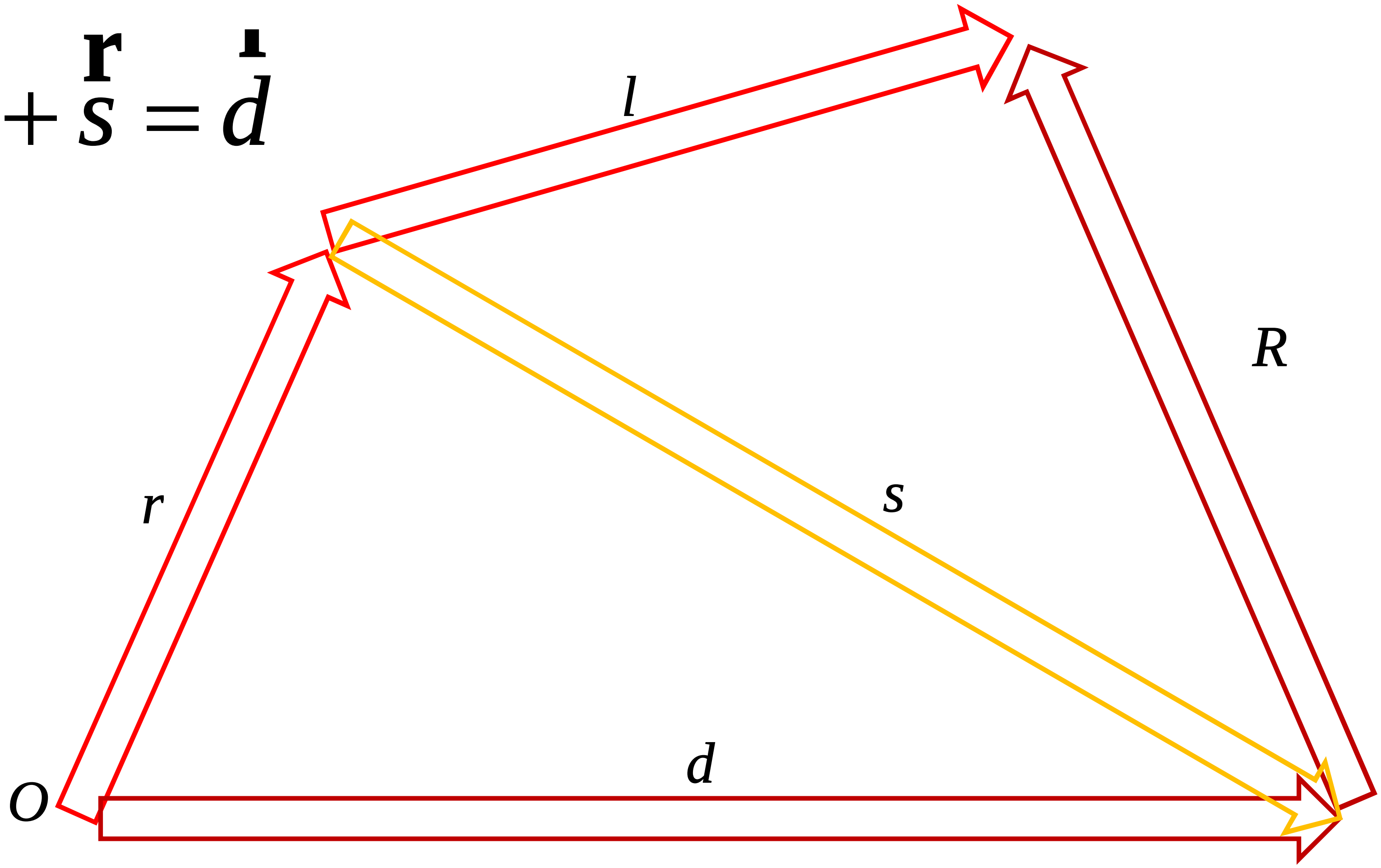
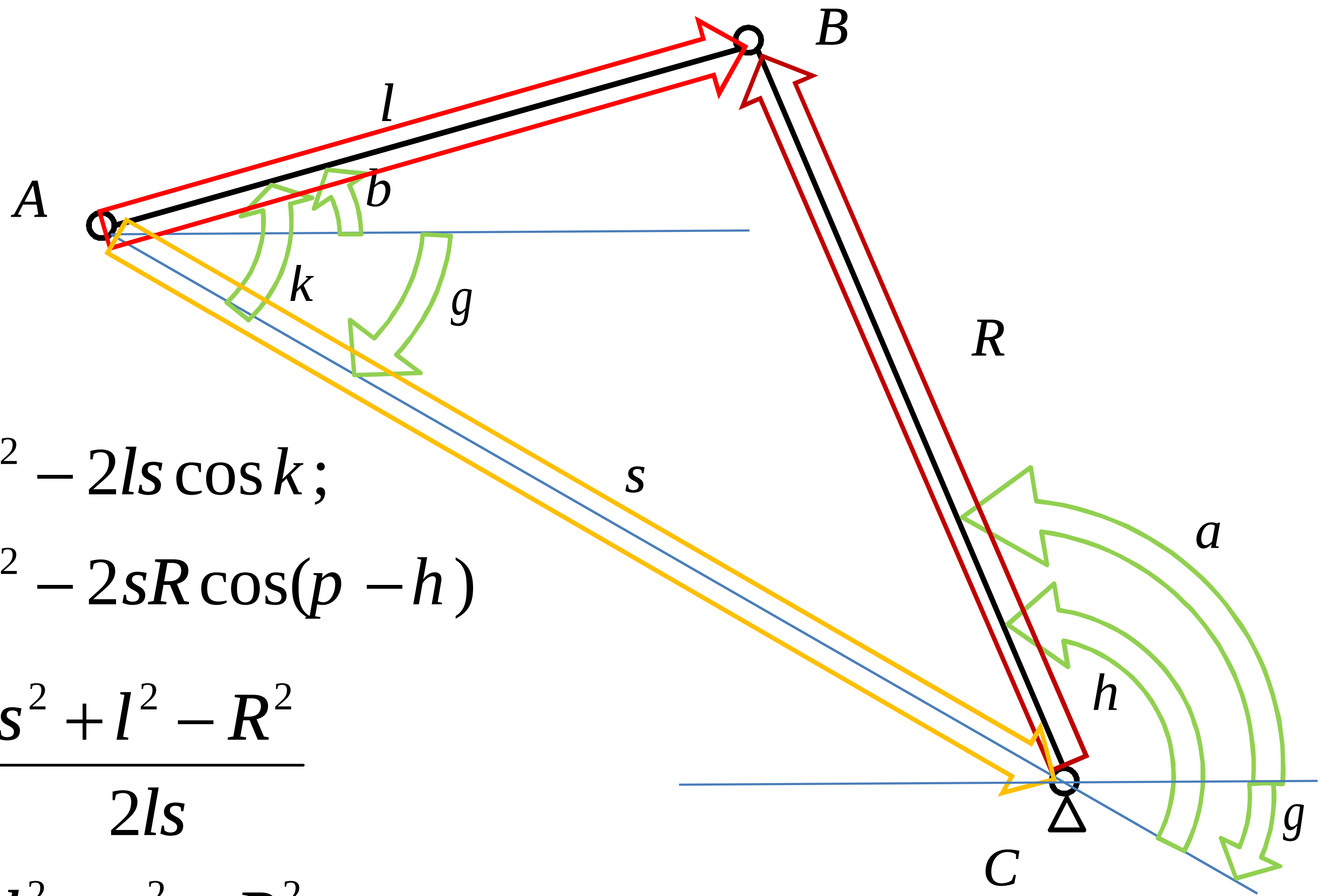
j
h
O
d
g
C

$$\vec{r} + \vec{l} = \vec{d} + \vec{R}$$

$$\vec{r} + \vec{s} = \vec{d}$$



$Om\ \Delta ABC \Rightarrow$



$$R^2 = s^2 + l^2 - 2ls\cos k;$$

$$l^2 = s^2 + R^2 - 2sR\cos(p - h)$$

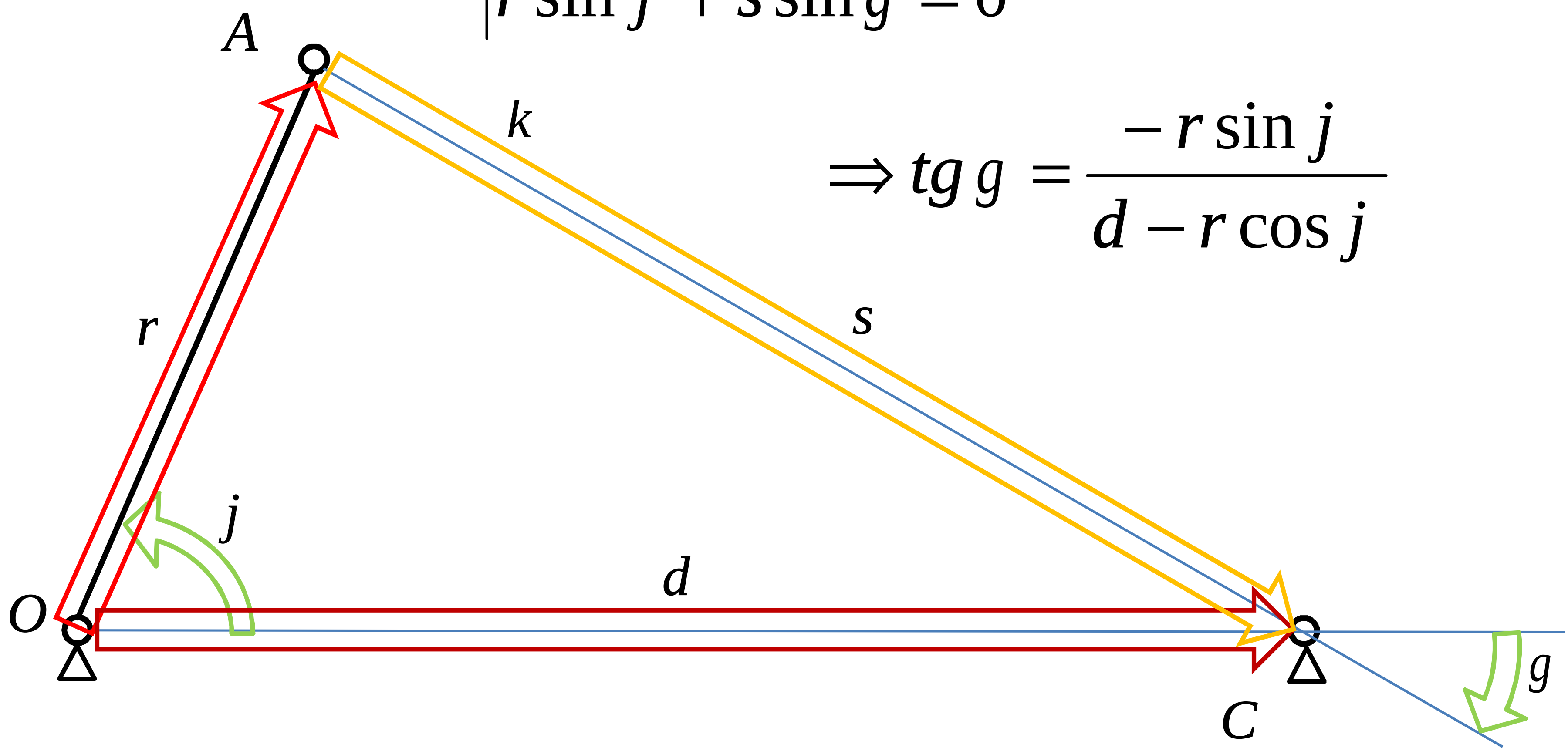
$$k = \arccos\frac{s^2 + l^2 - R^2}{2ls}$$

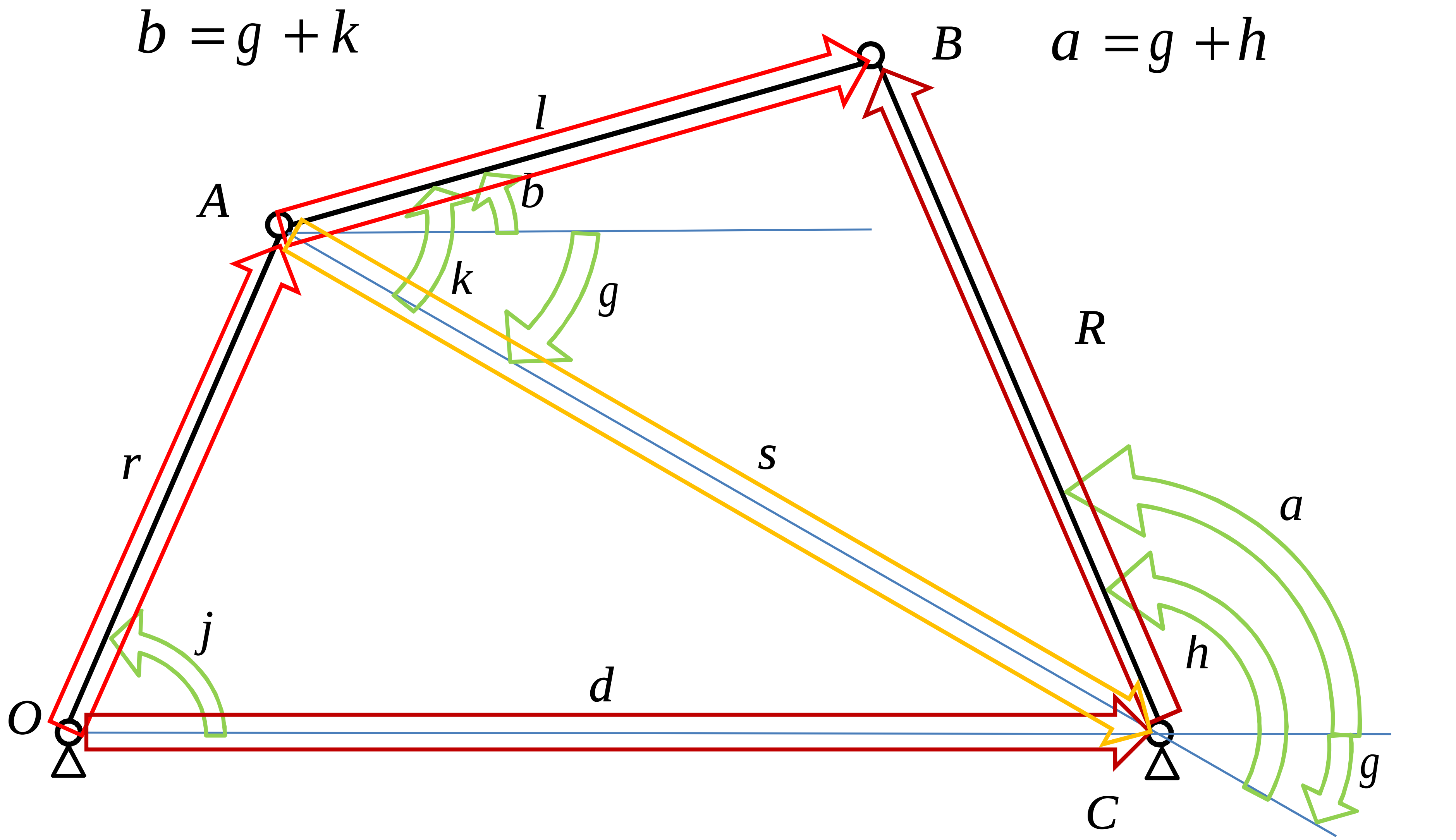
$$h = \arccos\frac{l^2 - s^2 - R^2}{2sR}$$

$$Om\ \Delta OAB \Rightarrow \quad s=\sqrt{r^{2}+d^{2}-2rd\cos j}$$

$$\left| \begin{array}{l} r\cos j + s\cos g = d \\ r\sin j + s\sin g = 0 \end{array} \right.$$

$$\Rightarrow tg\, g = \frac{-r\sin j}{d-r\cos j}$$





b = g + k
a = g + h
B
l
A
b
k
g
R
r
s
a
j
h
O
d
g
C

$$b = arctg\frac{-r\sin j}{d - r\cos j} + \arccos\frac{s^2 + l^2 - R^2}{2ls}$$

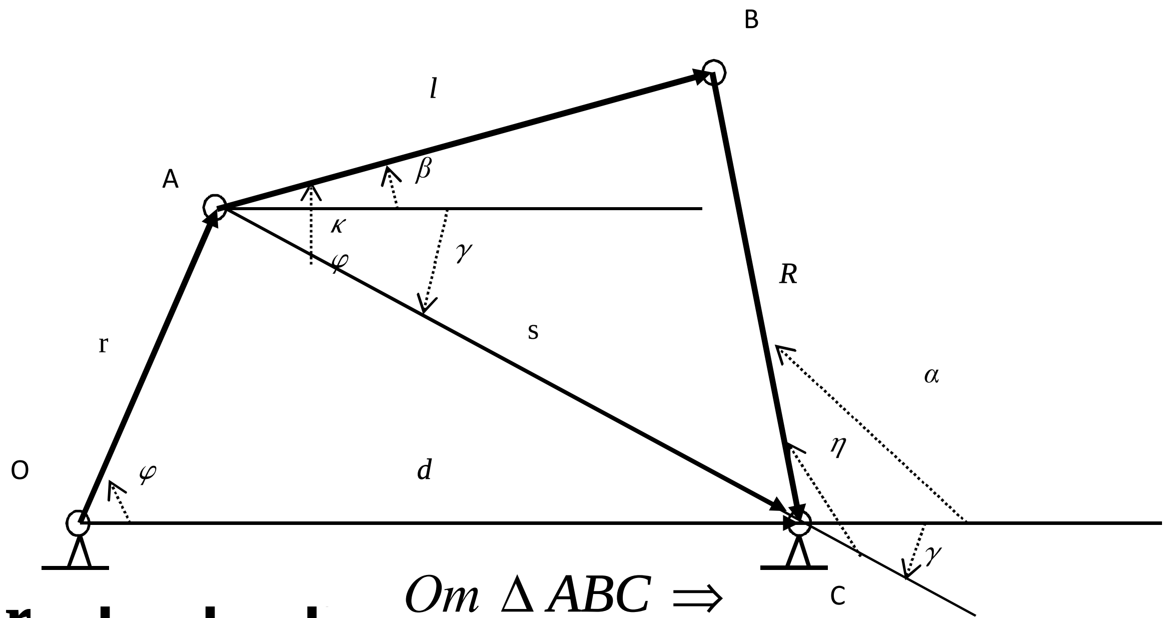
$$a = arctg\frac{-r\sin j}{d - r\cos j} + \arccos\frac{l^2 - s^2 - R^2}{2sR}$$

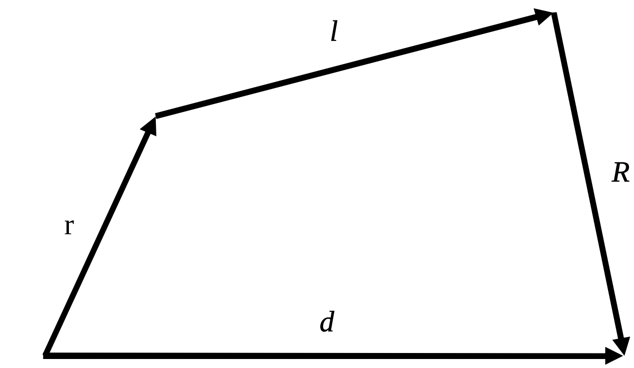
$$w_3 = \frac{da}{dj}\frac{dj}{dt} = \frac{da}{dj}w_1$$

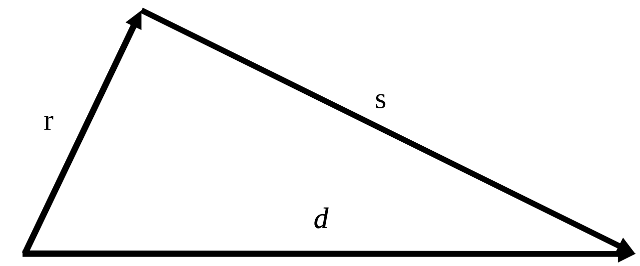
$$w_2 = \frac{db}{dj}\frac{dj}{dt} = \frac{db}{dj}w_1$$

$$\frac{db}{dj}=\frac{\dfrac{d\,r\,Sin\,j}{l\sqrt{d^2+r^2-2d\,rCos\,j}}-\dfrac{d\,r\left(d^2+l^2+r^2-R^2-2\,d\,r\,Cos\,j\right)Sin\,j}{2l\left(\sqrt{d^2+r^2-2d\,rCos\,j}\right)^3}}{\sqrt{1-\dfrac{\left(d^2+l^2+r^2-R^2-2\,d\,r\,Cos\,j\right)^2}{4l^2\left(d^2+r^2-2\,d\,r\,Cos\,j\right)}}}-$$

$$-\frac{\dfrac{r\;Cos\,j}{d-r\;Cos\,j}-\dfrac{r^2Sin^2j}{\left(d-r\;Cos\,j\right)^2}}{1+\dfrac{r^2Sin^2j}{\left(d-r\;Cos\,j\right)^2}}$$







$$\vec{r} + \vec{l} = \vec{d} + \vec{R}$$

$$\vec{r} + \vec{s} = \vec{d}$$

От $\Delta ABC \Rightarrow$

$$R^2 = s^2 + l^2 - 2ls\cos\kappa;$$

$$l^2 = s^2 + R^2 - 2sR\cos(\pi - \eta)$$

$$s = \sqrt{r^2 + d^2 - 2rd\cos\varphi}$$

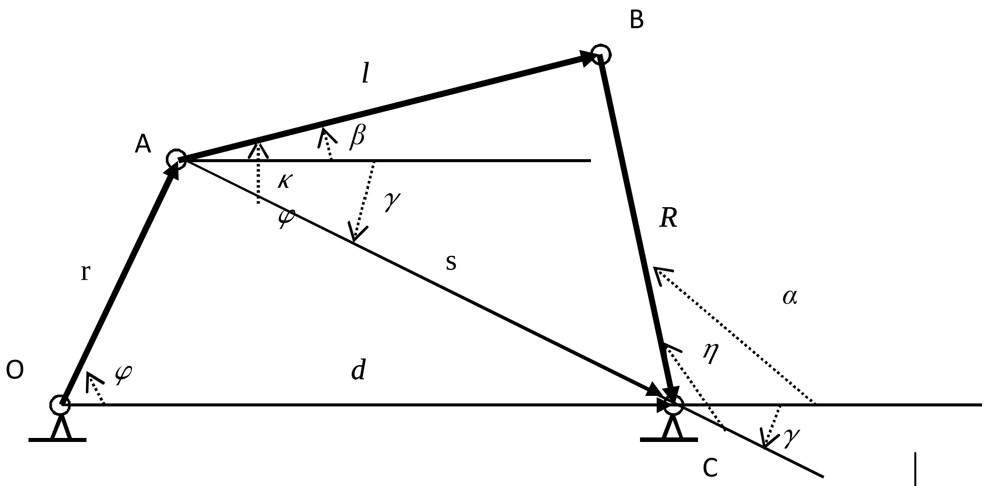
$$\beta = \gamma + \kappa \qquad \beta = arctg\frac{-r\sin\varphi}{d - r\cos\varphi} + \arccos\frac{s^2 + l^2 - R^2}{2ls}$$

$$\left| \begin{array}{l} r\cos\varphi + s\cos\gamma = d \\ r\sin\varphi + s\sin\gamma = 0 \end{array} \right.$$

$$\alpha = \gamma + \eta \qquad \alpha = arctg\frac{-r\sin\varphi}{d - r\cos\varphi} + \arccos\frac{l^2 - s^2 - R^2}{2sR}$$

$$\eta = \arccos\frac{l^2 - s^2 - R^2}{2sR} \qquad \kappa = \arccos\frac{s^2 + l^2 - R^2}{2ls}$$

$$\Rightarrow tg\,\gamma = \frac{-r\sin\varphi}{d - r\cos\varphi}$$



$$\vec{r} + \vec{l} = \vec{d} + \vec{R}$$

$$\begin{cases} r\cos j + l\cos b = d + R\cos a \\ r\sin j + l\sin b = R\sin a \end{cases}$$

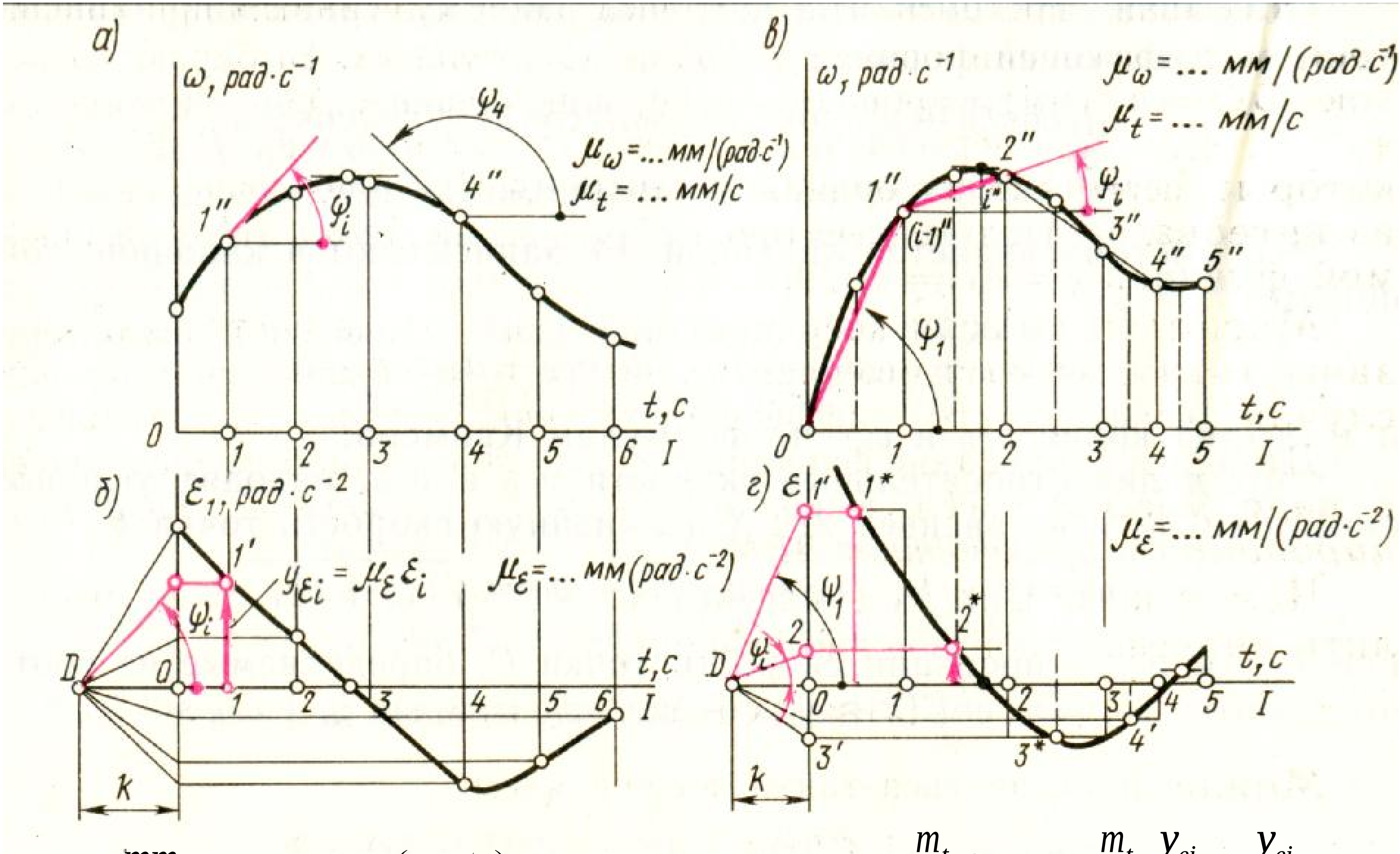
$$\begin{cases} -r\sin j - l\sin b \dfrac{db}{dj} = -R\sin a \dfrac{da}{dj} \\ r\cos j + l\cos b \dfrac{db}{dj} = R\cos a \dfrac{da}{dj} \end{cases}$$

$$\begin{cases} -r\sin j - l\sin b \dfrac{db/dt}{dj/dt} = -R\sin a \dfrac{da/dt}{dj/dt} \\ r\cos j + l\cos b \dfrac{db/dt}{dj/dt} = R\cos a \dfrac{da/dt}{dj/dt} \end{cases}$$

$$\begin{cases} -r\sin j - l\sin b \dfrac{w_2}{w_1} = -R\sin a \dfrac{w_3}{w_1} \\ r\cos j + l\cos b \dfrac{w_2}{w_1} = R\cos a \dfrac{w_3}{w_1} \end{cases}$$

$$\begin{cases} \dfrac{w_2}{w_1} = i_{21} = \dfrac{r\sin(j - a)}{l\sin(a - b)} \\ \dfrac{w_3}{w_1} = i_{31} = \dfrac{r\sin(j - b)}{R\sin(a - b)} \end{cases}$$

Графично и числено диференциране



$$m_w = \frac{mm}{rad/sek} \qquad e = \frac{dw}{dt} = \frac{d\left(y_w/m_w\right)}{d\left(x_t/m_t\right)} = \frac{m_t}{m_w}\frac{dy_w}{dx_t} = \frac{m_t}{m_w} tgy$$

$$m_t = \frac{mm}{sek}$$

$$e = \frac{m_t}{m_w} tgy_i = \frac{m_t}{m_w}\frac{y_{ei}}{K} = \frac{y_{ei}}{m_e}$$

$$tgy_i = \frac{y_{ei}}{K} \qquad m_e = \frac{m_w K}{m_t}$$

Методи за числено диференциране на функция зададена с масив от числа:

$$f'(x) \approx \left[f(x+\Delta x) - f(x) \right] / \Delta x$$

$$f'(x)_{i+1} \approx \frac{1}{2} \left\{ \frac{\left[f(x)_{i+1} - f(x)_i \right]}{\Delta x_i} + \frac{\left[f(x)_{i+2} - f(x)_{i+1} \right]}{\Delta x_{i+1}} \right\}$$

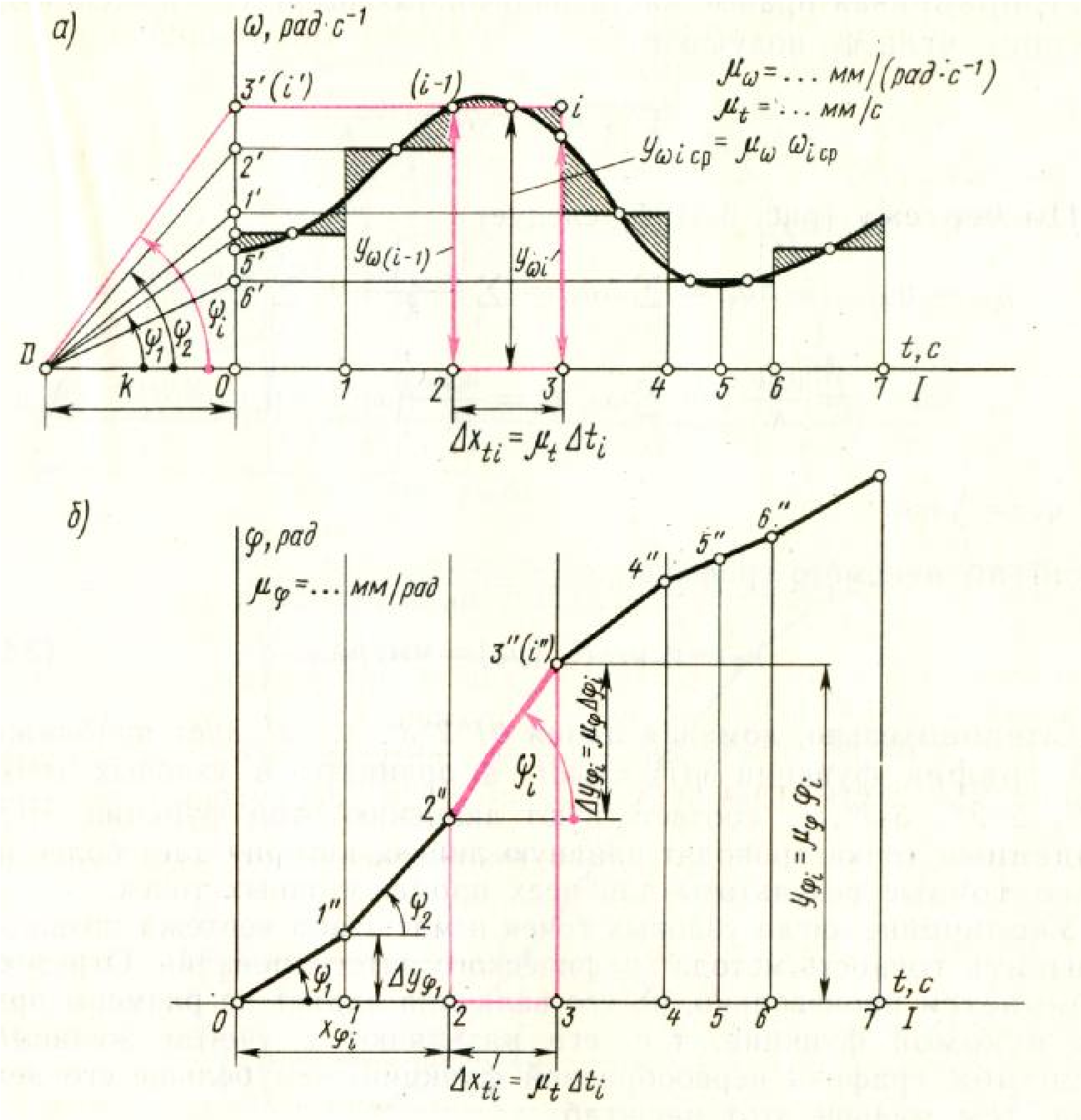
Методи за числено диференциране на функция зададена с таблица от числа за равноотстоящи стоисти на аргумента:

$$x_i = x_0 + i.\Delta x \; (i = 0; \pm 1; \pm 2; \ldots),$$

$$y_i' = y'(x_i) = \tfrac{1}{\Delta x} \left(\Delta y_i - \tfrac{1}{2} \Delta^2 y_i + \tfrac{1}{3} \Delta^3 y_i - \ldots \right);$$

$$y_i'' = y''(x_i) = \tfrac{1}{(\Delta x)^2} \left(\Delta^2 y_i - \tfrac{1}{2} \Delta^3 y_i + \tfrac{11}{12} \Delta^4 y_i - \ldots \right).$$

Графично и числено интегриране



$$m_j = \frac{m_w m_t}{K}$$

Формули на Нютон, Гаус, Симпсон и др.

При зададени значения на функцията $y_i = y(x_i)$

за $n+1$ равноотдалечени значения на аргумента

$$x_i = x_0 + i.\Delta x_i \quad (i = 0, 1, 2, \ldots n)$$

$$Newton \Rightarrow \mathrm{I} = \int_{x}^{x+n\Delta x} y(x)dx \approx \Delta x\left(\tfrac{1}{2}y_0 + y_1 + y_2 + \ldots + y_{n-1} + \tfrac{1}{2}y_n\right)$$

$$Трапеца \Rightarrow \mathrm{I} = \frac{\Delta x}{2}(y_0 + y_1)$$

$$Симпсон \Rightarrow \mathrm{I} = \frac{\Delta x}{3}(y_0 + 4y_1 + y_2)$$