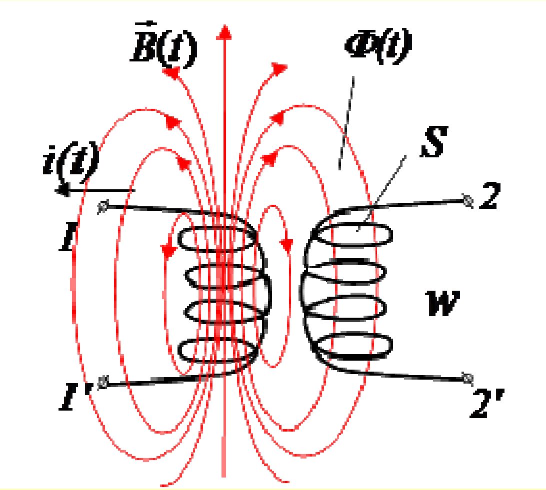
Вериги с индуктивни връзки

Във веригата са включени **магнитносвързани бобини**, т.е. преминаването на ток през едната създава магнитно поле, което обхваща навивките на другата и съгласно закона за електромагнитната индукция **индуктира** в нея **напрежение**.



$$i(t) \longrightarrow \vec{B}(t)$$

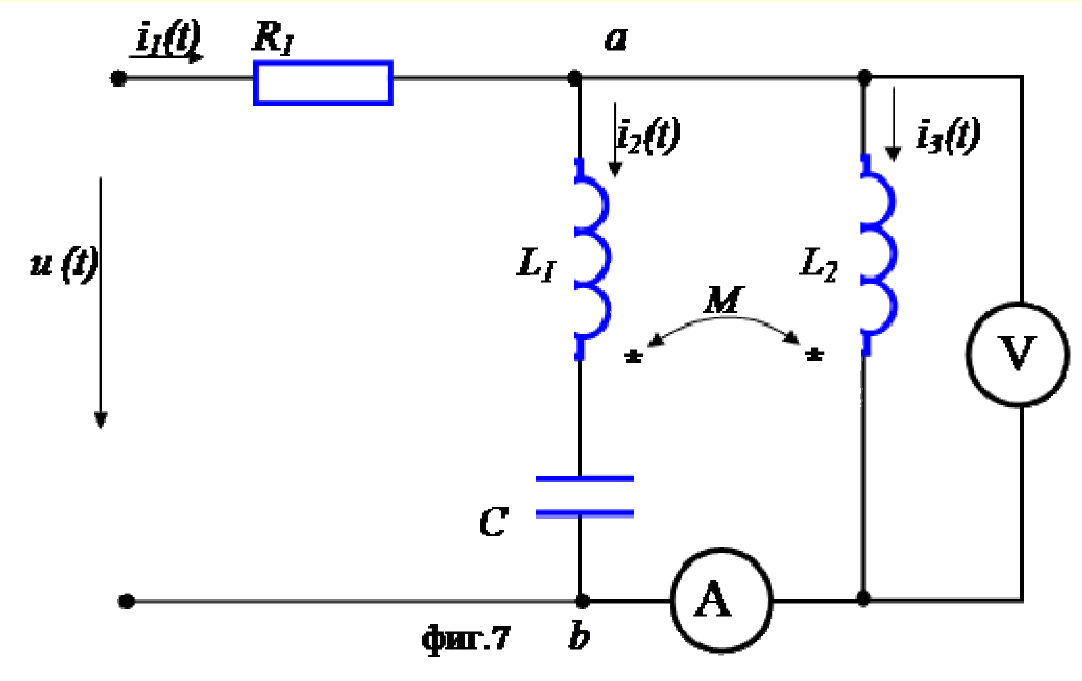
$$\Phi(t) = \oiint_S \vec{B}(t)d\vec{S}$$

$$\Psi(t) = w.\Phi(t)$$

$$e(t) = -\frac{d\Psi}{dt}$$

Вериги с индуктивни връзки

- Вериги, с включени в тях бобини, между които има индуктивни връзки се наричат **вериги с индуктивни връзки**



- преминаването на **ток през едната** води до появата на **напрежение в другата** и обратно

- при анализа на вериги с индуктивни връзки се отчитат и тези допълнителни напрежения

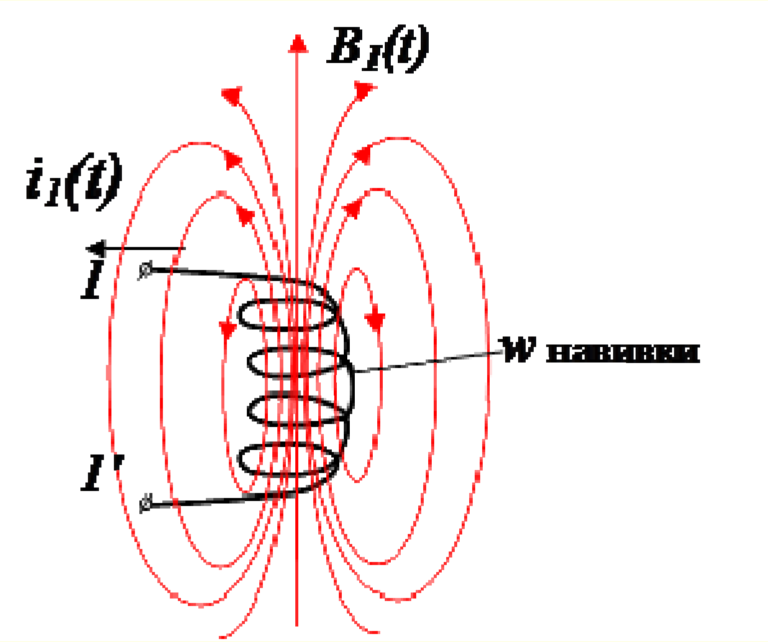
<u>ЕДН на самоиндукция</u>

през бобината ***"1"*** преминава променлив ток

$i_1(t) \longrightarrow B_1(t)$

$$\Phi_1(t) = \oiint_S B_1(t)dS$$

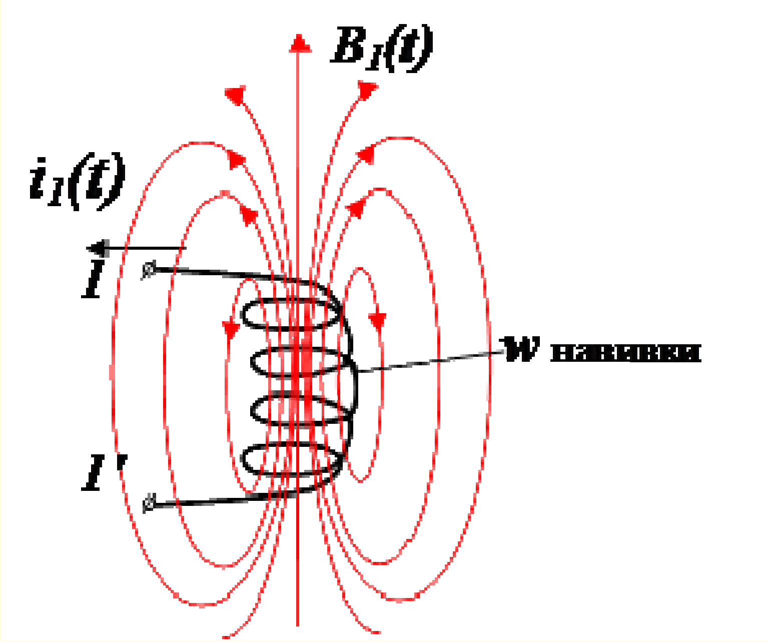
$$\Psi_1(t) = w.\Phi_1(t)$$



$$e_1(t) = -\frac{d\Psi_1}{dt}$$

- *е.д.н.* на самоиндукция е <u>пропорционално на скоростта на изменение на тока</u>
- <u>Знакът ***"-"*** *е.д.н.* на самоиндукция се противопоставя на изменението на тока</u>

ЕДН на самоиндукция



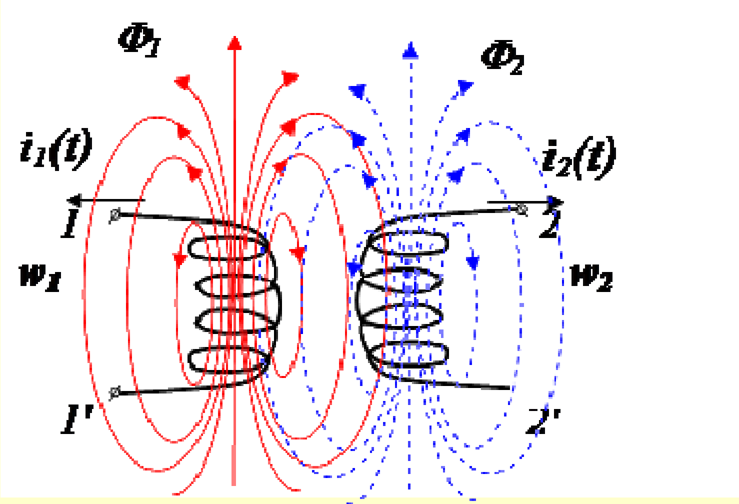
Връзката между потока $\Psi(t)$ и тока $i(t)$, се определя с параметъра L - ***собствена индуктивност***.

$$\Psi_1 = L_1 i_1$$

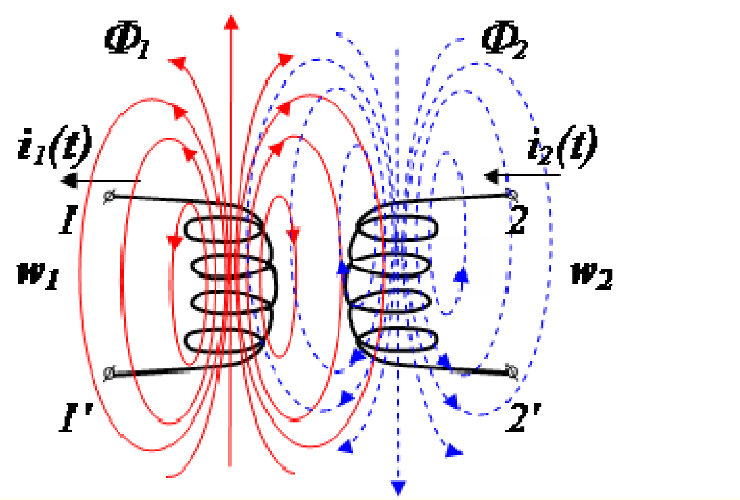
- ***L*** зависи от геометрията и характеристиките на средата
- ***L*** не зависи от големината на тока

$$e_1(t) = -\frac{d\Psi_1}{dt} = -\frac{d}{dt}(L_1 . i) \quad \Rightarrow \boxed{e_1(t) = -L_1 \frac{di}{dt}}$$

ЕДН на взаимоиндукция



$i_1(t) \longrightarrow \Phi_1(t) = \Phi_{11}(t) + \Phi_{21}(t)$



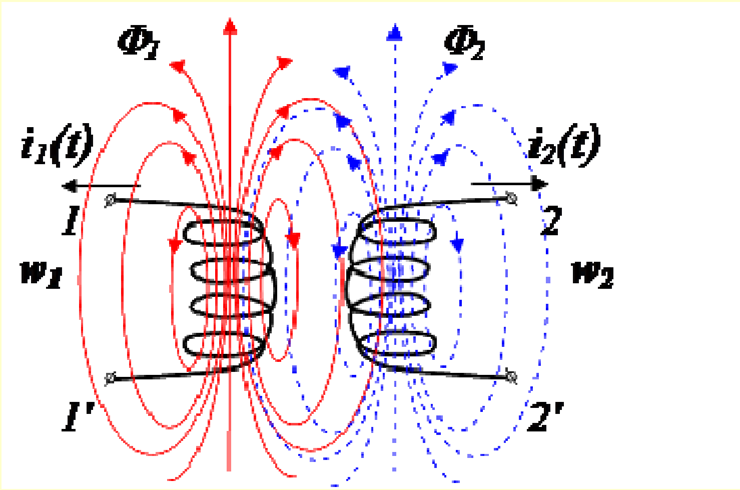
$i_1(t) \longrightarrow \Phi_1(t) = \Phi_{11}(t) + \Phi_{21}(t)$

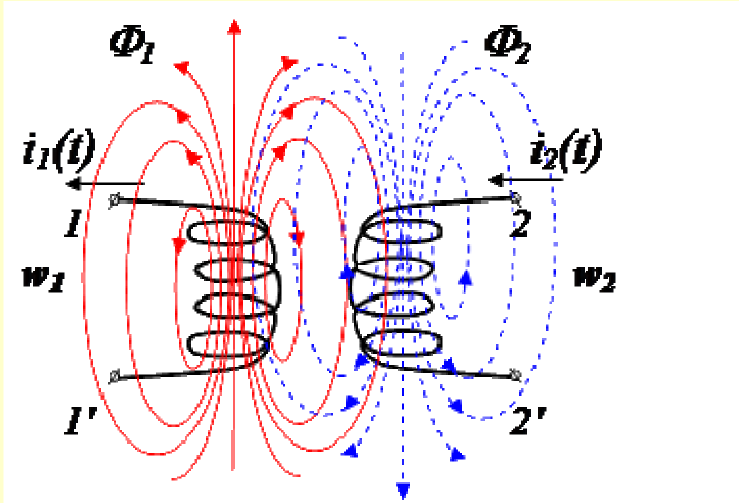
$i_2(t) \longrightarrow \Phi_2(t) = \Phi_{22}(t) + \Phi_{12}(t)$

$$\Psi_1 = \Psi_{11} - \Psi_{12} = w_1 \cdot \Phi_{11} - w_1 \cdot \Phi_{12}$$

$$\Psi_2 = \Psi_{22} - \Psi_{21} = w_2 \cdot \Phi_{22} - w_2 \cdot \Phi_{21}$$

ЕДН на взаимоиндукция





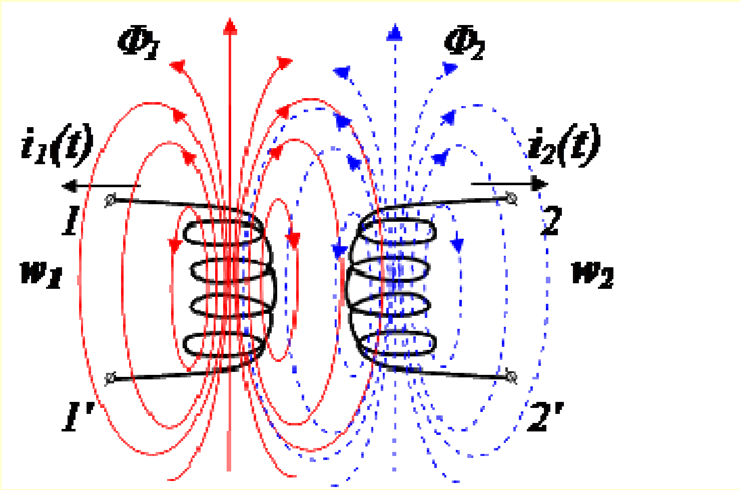
$$\Psi_1 = \Psi_{11} \pm \Psi_{12} = w_1.\Phi_{11} \pm w_1.\Phi_{12}$$

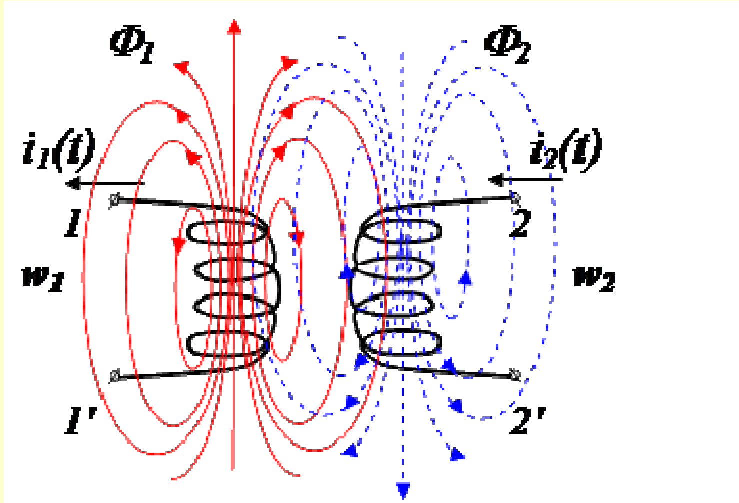
$$e_1(t) = -\frac{d\Psi_1}{dt} = -\frac{d}{dt}(\Psi_{11} \pm \Psi_{12}) = -\frac{d}{dt}(L_1 i_1 \pm M_{12} i_2) = -(L_1 \frac{di_1}{dt} \pm M_{12} \frac{di_2}{dt})$$

$$e_{L_1}(t) = -L_1 \frac{di_1}{dt}$$

$$e_{M_1}(t) = \pm M_{12} \frac{di_2}{dt}$$

ЕДН на взаимоиндукция





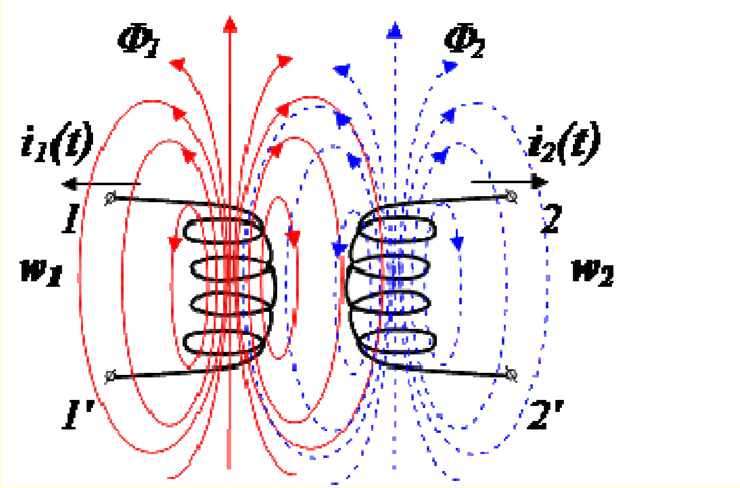
$$\Psi_2 = \Psi_{22} \pm \Psi_{21} = w_2.\Phi_{22} \pm w_2.\Phi_{21}$$

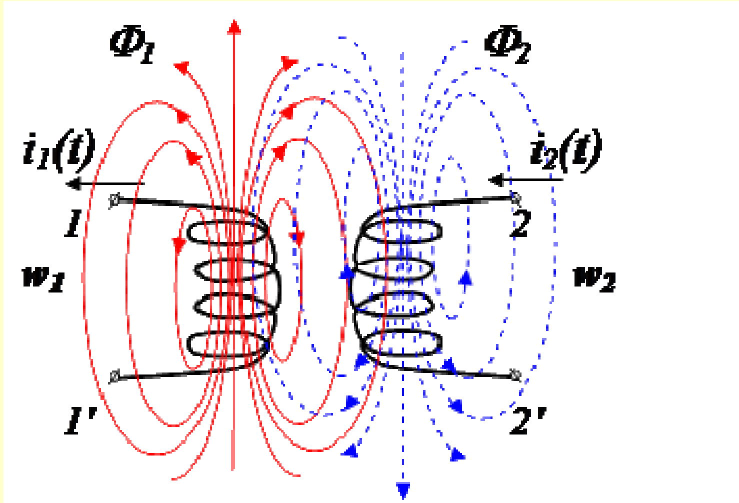
$$e_2(t) = -\frac{d\Psi_2}{dt} = -\frac{d}{dt}(\Psi_{22} \pm \Psi_{21}) = -\frac{d}{dt}(L_2 i_2 \pm M_{21} i_1) = -(L_2 \frac{di_2}{dt} \pm M_{21} \frac{di_1}{dt})$$

$$e_{L_2}(t) = -L_2 \frac{di_2}{dt}$$

$$e_{M_2}(t) = \pm M_{21} \frac{di_1}{dt}$$

ЕДН на взаимоиндукция





За бобина "1"

$$e_1(t) = L_1 \frac{di_1}{dt} \pm M_{12} \frac{di_2}{dt}$$

За бобина "2"

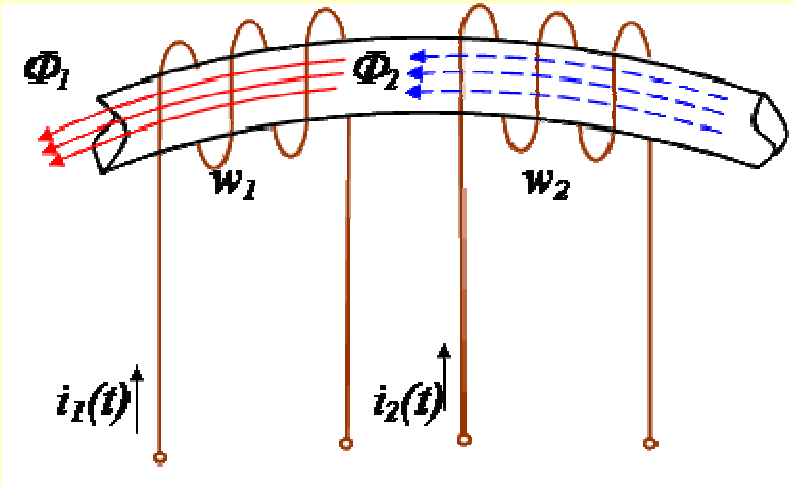
$$e_2(t) = L_2 \frac{di_2}{dt} \pm M_{21} \frac{di_1}{dt}$$

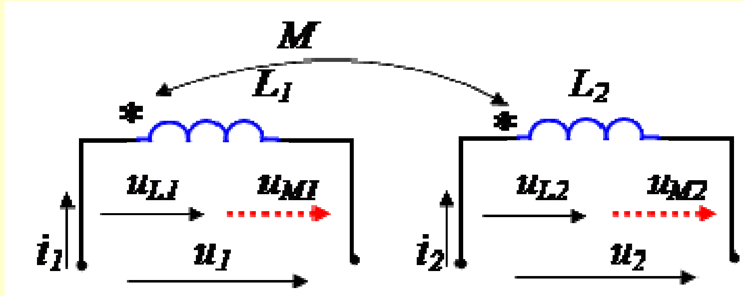
е.д.н. на самоиндукция

***е.д.н.* на взаимоиндукция**

$$M_{12} = M_{21} = M$$

Едноименни изводи.
Определяне знака на напрежението от взаимна индукция





Съгласувано свързване

Собственият и взаимният магн.поток имат една и съща посока

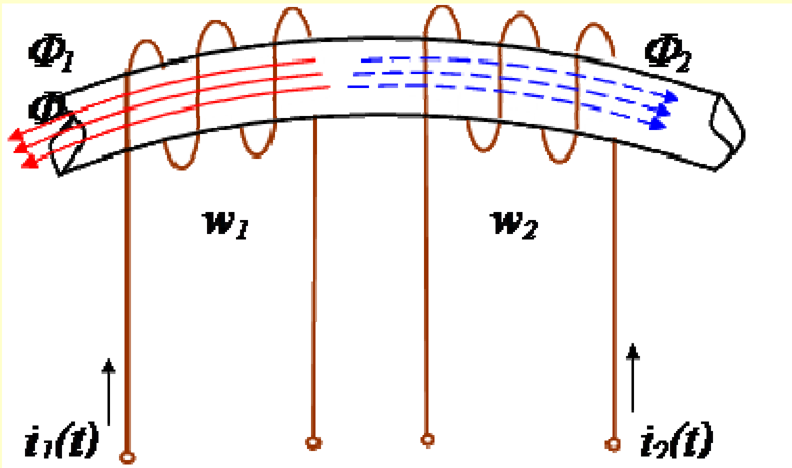
Токовете са еднакво ориентирани спрямо едноименните изводи

Напрежението е сума от *е.д.н.* на самоиндукция и *е.д.н.* на взаимоиндукция

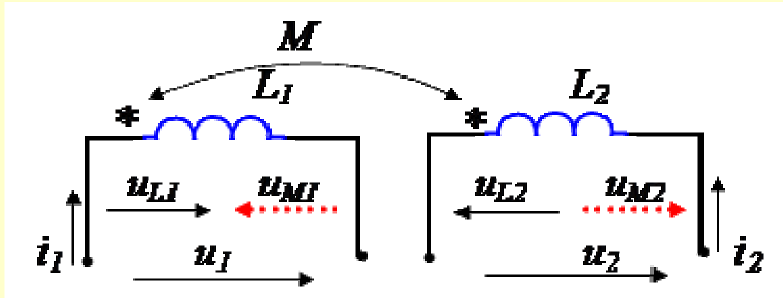
$$u_1 = u_{L_1} + u_{M_1}; \qquad u_2 = u_{L_2} + u_{M_2}$$

$$u_1(t) = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}; \qquad u_2(t) = L_2\frac{di_2}{dt} + M\frac{di_1}{dt}$$

Определяне знака на напрежението от взаимна индукция



$$u_1 = u_{L_1} - u_{M_1}$$
$$u_2 = u_{L_2} - u_{M_2}$$



Несъгласувано свързване

Собственият и взаимният магн.поток имат противоположни посоки

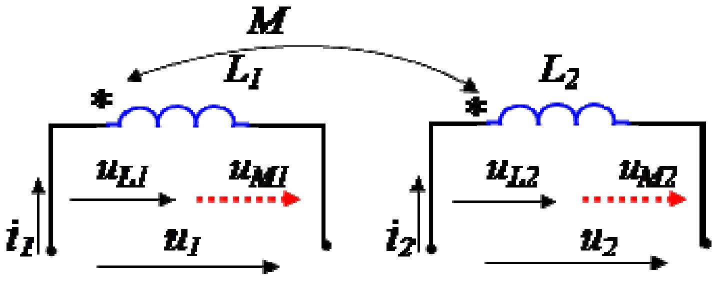
Токовете са различно ориентирани спрямо едноименните изводи

Напрежението е **разлика** от *е.д.н.* на самоиндукция и *е.д.н.* на взаимоиндукция

$u_1 = u_{L_1} - u_{M_1};$ $\qquad$ $u_2 = u_{L_2} - u_{M_2}$

$u_1(t) = L_1 \dfrac{di_1}{dt} - M \dfrac{di_2}{dt};$ $\qquad$ $u_2(t) = L_2 \dfrac{di_2}{dt} - M \dfrac{di_1}{dt}$

Определяне знака на напрежението от взаимна индукция

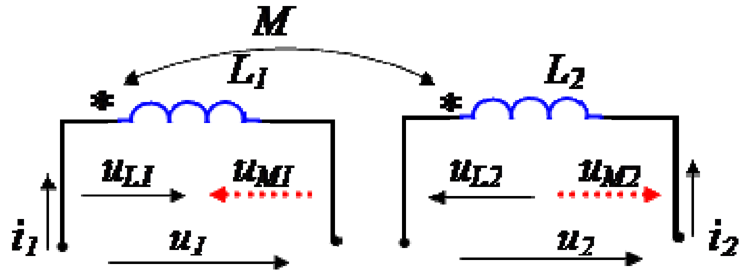


$$u_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$u_2(t) = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$\dot{U}_1 = j\omega L_1.\dot{I}_1 + j\omega M.\dot{I}_2 = Z_{L_1}.\dot{I}_1 + Z_M.\dot{I}_2$$

$$\dot{U}_2 = j\omega L_2.\dot{I}_2 + j\omega M.\dot{I}_1 = Z_{L_2}.\dot{I}_2 + Z_M.\dot{I}_1$$



$$u_1(t) = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

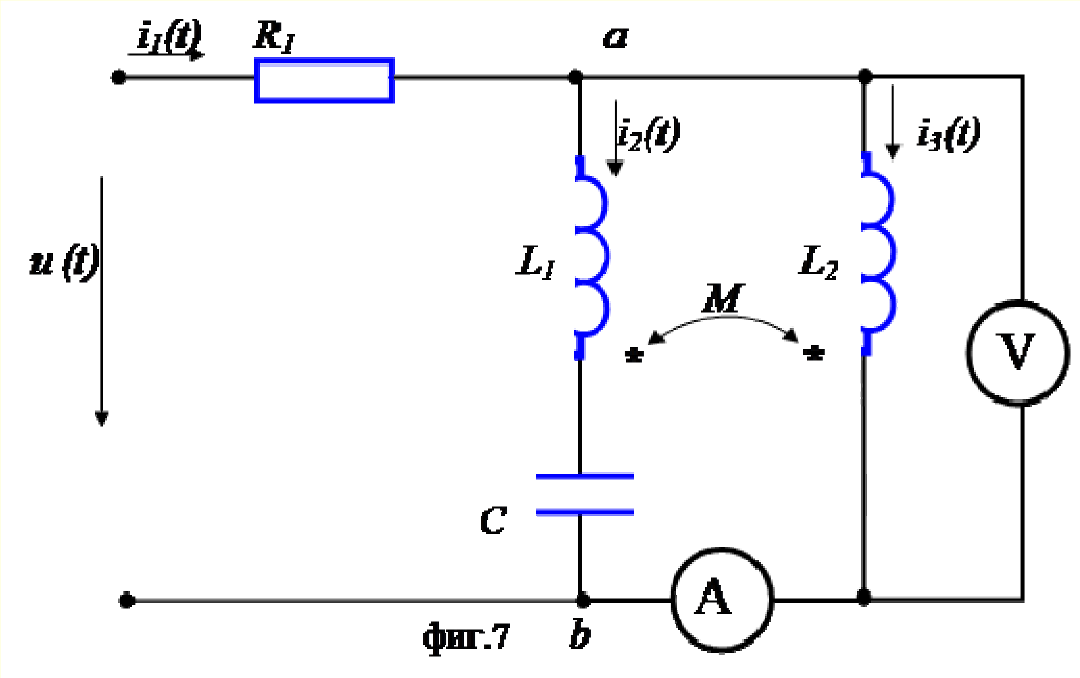
$$u_2(t) = L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$

$$\dot{U}_1 = j\omega L_1.\dot{I}_1 - j\omega M.\dot{I}_2 = Z_{L_1}.\dot{I}_1 - Z_M.\dot{I}_2$$

$$\dot{U}_2 = j\omega L_2.\dot{I}_2 - j\omega M.\dot{I}_1 = Z_{L_2}.\dot{I}_2 - Z_M.\dot{I}_1$$

Пример за анализ на верига с индуктивни връзки:

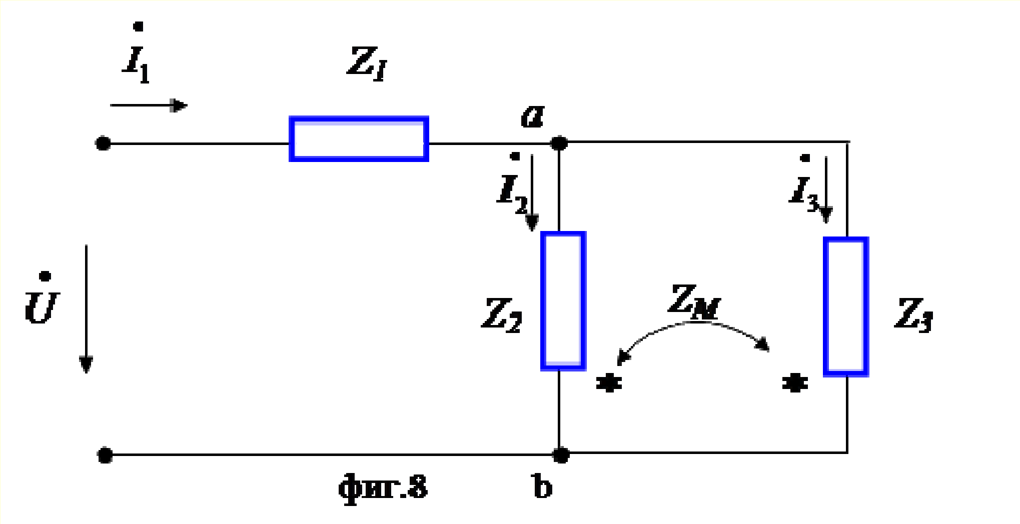
Да се определят токовете $i_1(t)$, $i_2(t)$ и $i_3(t)$ и показанията на уредите.



$u(t) = 141\sin(\omega t+90)V$

$f=160Hz$,
$L_1=40\ mH$,
$L_2=30\ mH$,
$M=10\ mH$,
$C=100\ \mu F$
$R_1 = 10\Omega$,

Решение



$$u(t) = 141 sin(\omega t + 90) V$$

$$\dot{U} = Ue^{j\psi_u} = \frac{u_m}{\sqrt{2}} e^{j\psi_u} = \frac{141}{\sqrt{2}} e^{j90}$$
$$= 100.[\cos(90) + j\sin(90)]$$
$$= 100.(0 + j) = j100V$$

$$\omega = 2\pi f = 2\pi.160 \approx 1000 = 10^3 rad/s$$

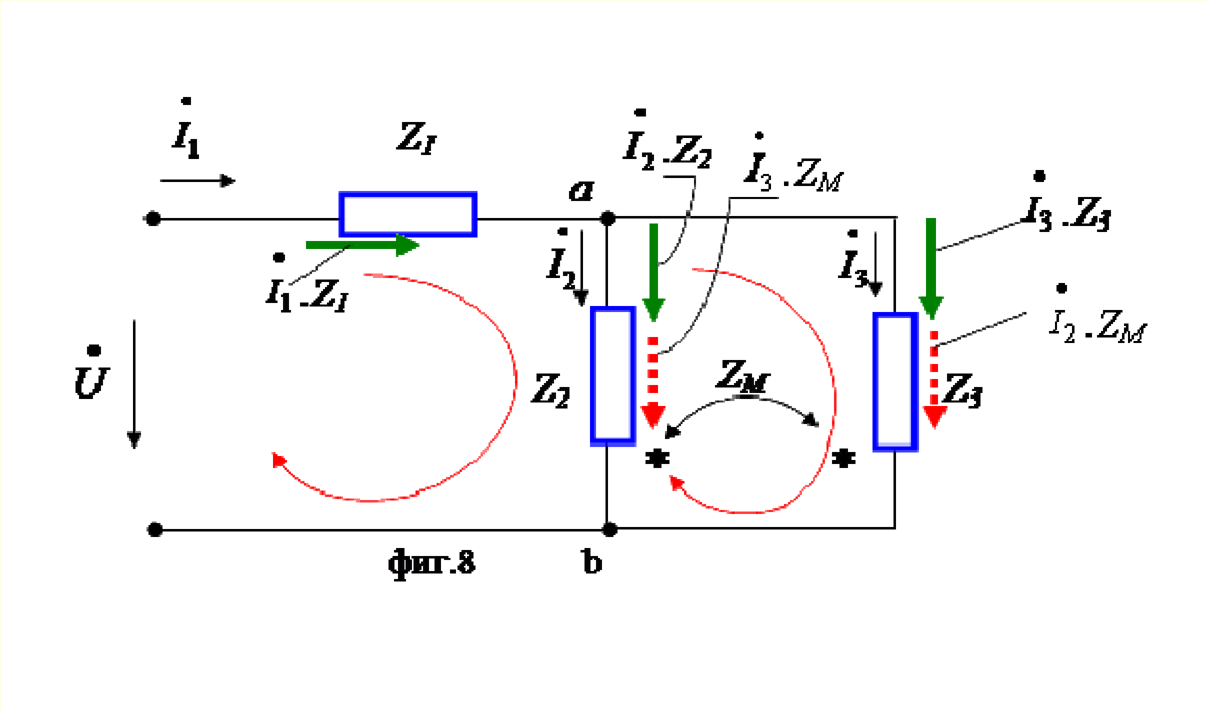
$$Z_1 = R_1 = 10\Omega$$

$$Z_2 = j\omega L_1 - j\frac{1}{\omega C} = j.10^3.40.10^{-3} - j\frac{1}{10^3.100.10^{-6}} = j(40-10) = j30\,\Omega$$

$$Z_3 = j\omega L_2 = j.10^3 30.10^{-3} = j30\,\Omega$$

$$Z_M = j\omega M = j.10^3 10.10^{-3} = j10\,\Omega$$

Решение



$n=2$

$m=3$

$$\dot{I}_1 - \dot{I}_2 - \dot{I}_3 = 0$$

$$\dot{I}_1 Z_1 + \dot{I}_2 Z_2 + \dot{I}_3 Z_M = \dot{U}$$

$$\dot{I}_3 Z_3 - \dot{I}_2 Z_2 + \dot{I}_2 Z_M - \dot{I}_3 Z_M = 0$$

Решение

$$\begin{cases} \dot{I}_1 - \dot{I}_2 - \dot{I}_3 = 0 \\ \dot{I}_1 10 + \dot{I}_2\, j30 + \dot{I}_3\, j10 = j100 \\ \dot{I}_3 .j30 - \dot{I}_2\, j30 + \dot{I}_2\, j10 - \dot{I}_3\, j10 = 0 \end{cases}$$

$$\begin{cases} \dot{I}_1 - \dot{I}_2 - \dot{I}_3 = 0 \\ \dot{I}_1 + \dot{I}_2\, j3 + \dot{I}_3\, j = j10 \\ \dot{I}_2 = \dot{I}_3 \end{cases}$$

Решение

5. Получаваме комплексите на трите тока:

$$\dot{I}_1 = (4 + j2) = 4{,}47e^{j26{,}56} A$$

$$\dot{I}_2 = (2 + j) = 2{,}24e^{j26{,}56} A$$

$$\dot{I}_3 = \dot{I}_2 = (2 + j) = 2{,}24e^{j26{,}56} A$$

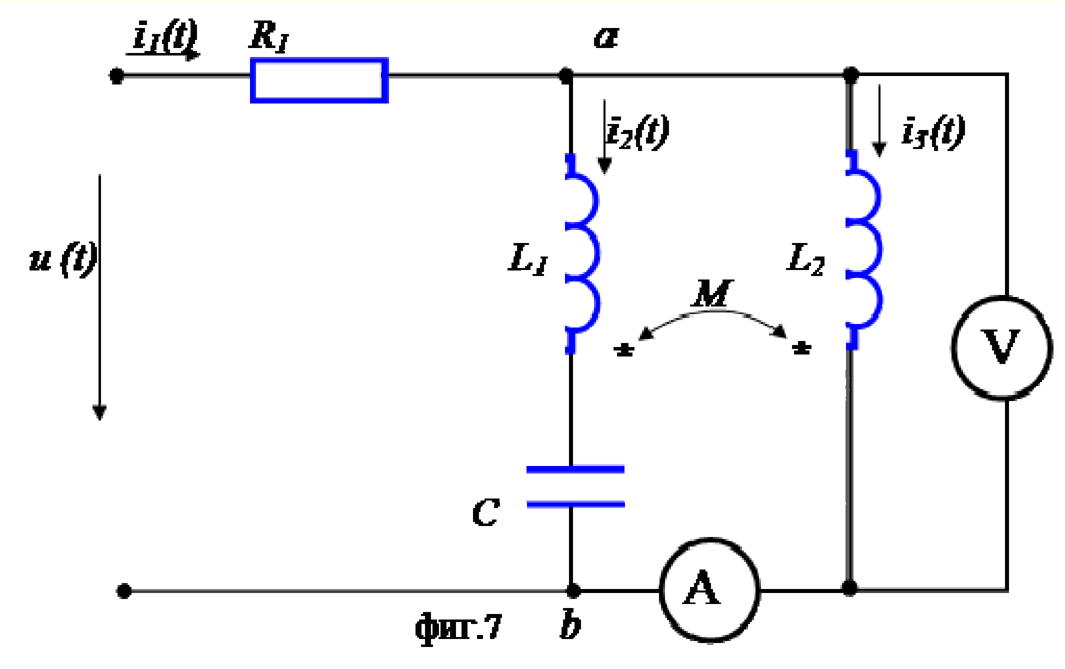
6. Тогава моментните стойности на токовете са:

$$i_1(t) = I_1\sqrt{2}\sin(\omega t + \psi_1) = 4{,}47\sqrt{2}\sin(1000t + 26{,}56^0)A$$

$$i_2(t) = I_2\sqrt{2}\sin(\omega t + \psi_2) = 2{,}24\sqrt{2}\sin(1000t + 26{,}56^0)A$$

$$i_3(t) = I_3\sqrt{2}\sin(\omega t + \psi_3) = 2{,}24\sqrt{2}\sin(1000t + 26{,}56^0)A$$

Показания на уредите:



$$\dot{I}_1 = (4 + j2) = 4,47e^{j26,56}\,A$$

$$I_A = I_3 = 2,24\,A$$

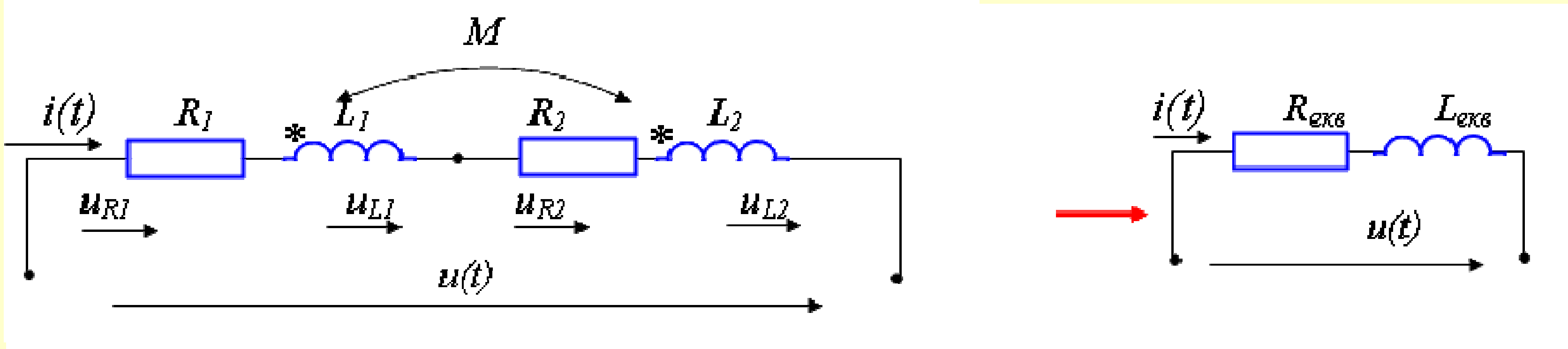
$$U_V = U_{ab}$$

$$\dot{U}_{ab} = Z_2.\dot{I}_2 + Z_M.\dot{I}_3 = j\omega L_2.\dot{I}_2 + j\omega M.\dot{I}_3 = (2+j)j30 + (2+j)j10 = (-40 + j80)V$$

$$\Rightarrow U_V = U_{ab} = \sqrt{(-40)^2 + 80^2} = 40\sqrt{5} = 89,44V$$

Последователно съединение на два индуктивно свързани елемента.

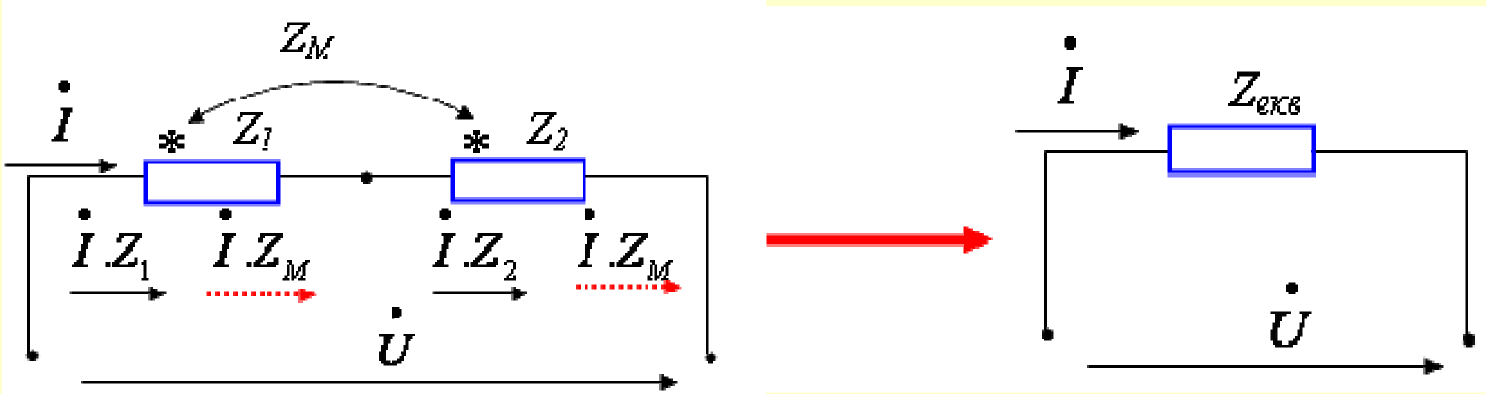
Съгласувано свързване



$$R_{екв} = R_1 + R_2$$

$$L_{екв} = L_1 + L_2 + 2M$$

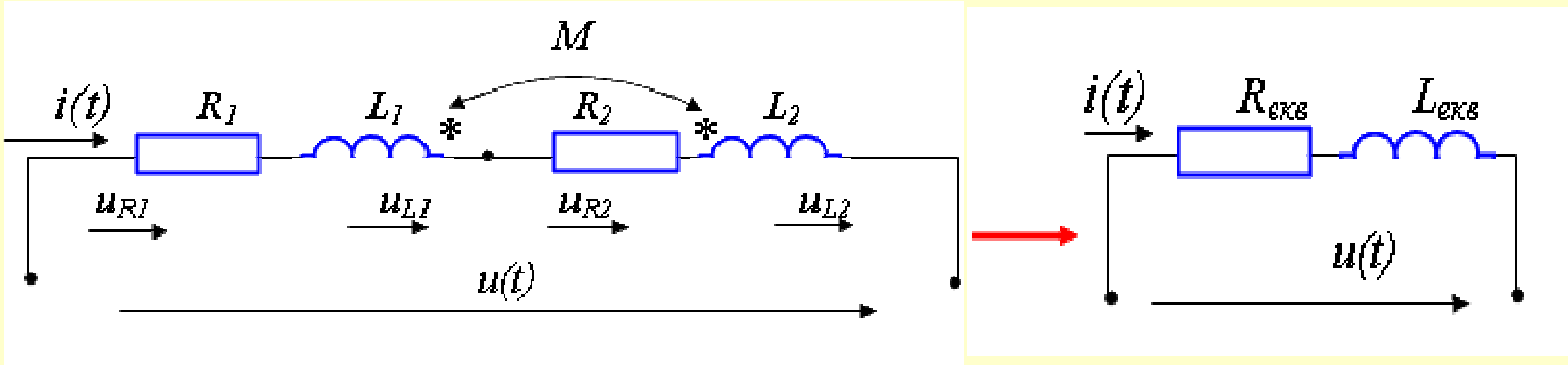
Съгласувано свързване



$$Z_1 = R_1 + j\omega L_1; \qquad Z_2 = R_2 + j\omega L_2; \qquad Z_M = j\omega M$$

$$Z_{екв} = Z_1 + Z_2 + 2Z_M$$

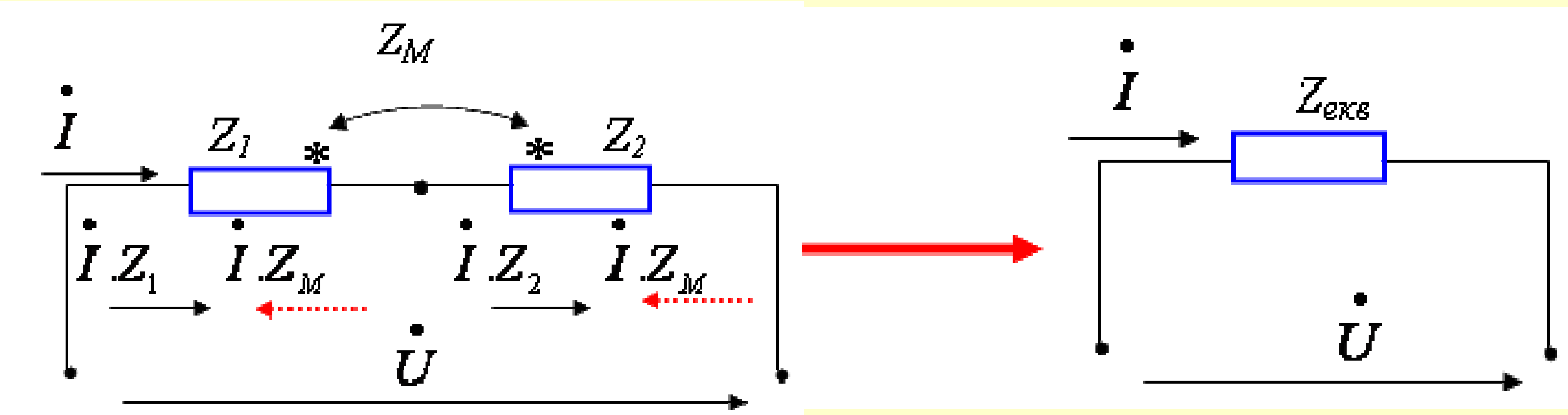
2. Несъгласувано свързване.



$$R_{екв} = R_1 + R_2$$

$$L_{екв} = L_1 + L_2 - 2M$$

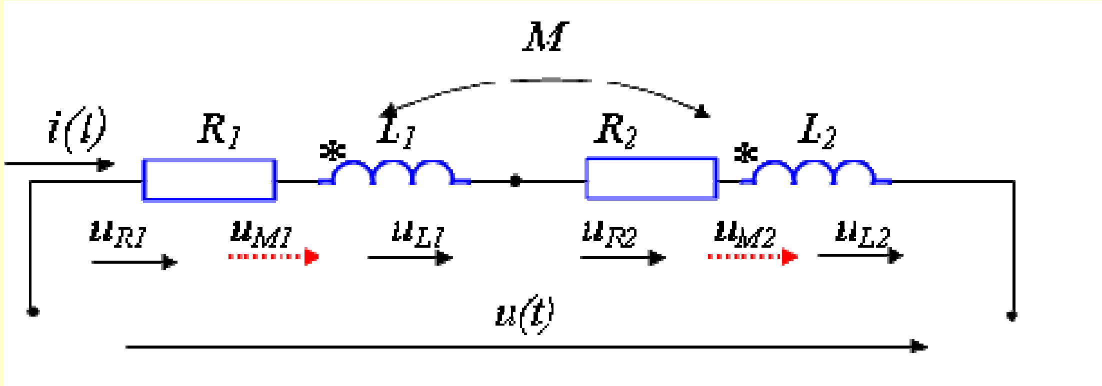
2. Несъгласувано свързване.

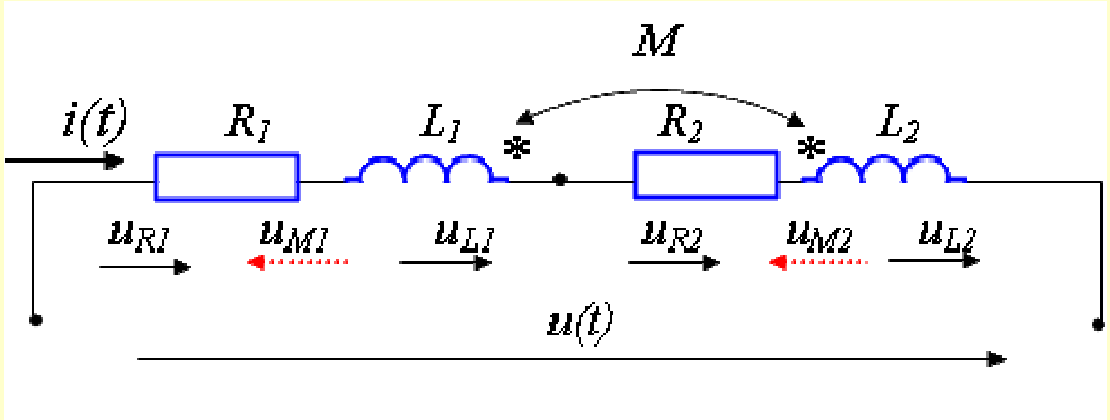


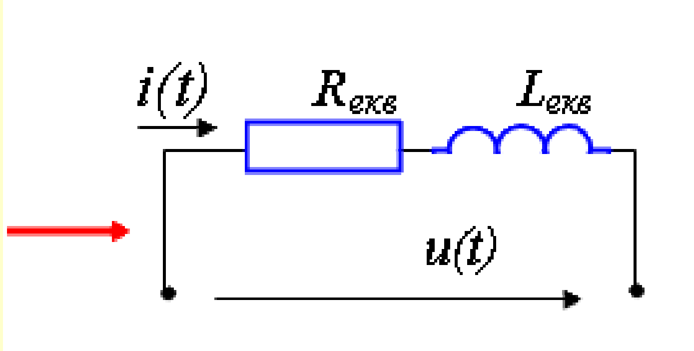
$$Z_1 = R_1 + j\omega L_1; \qquad Z_2 = R_2 + j\omega L_2; \qquad Z_M = j\omega M$$

$$Z_{екв} = Z_1 + Z_2 - 2Z_M$$

Извод



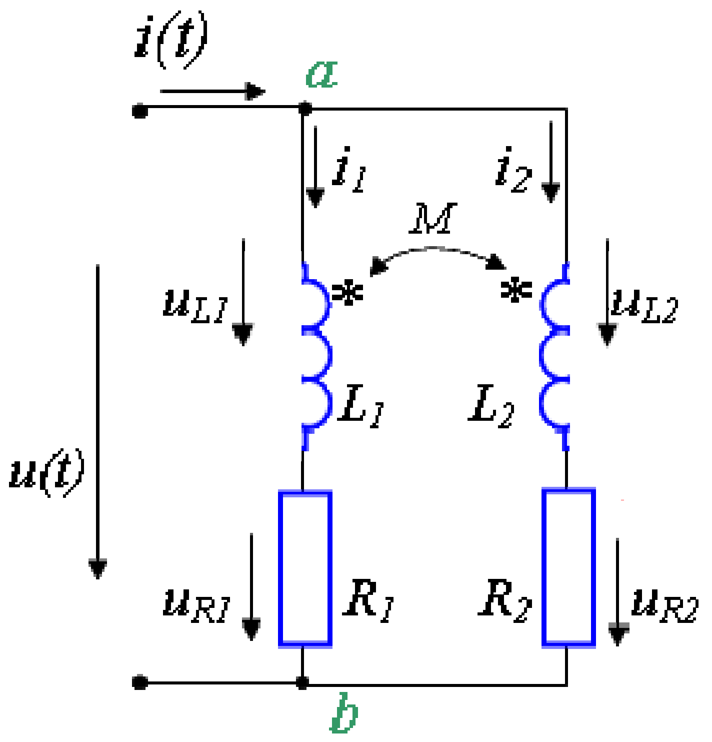




$$R_{екв} = R_1 + R_2$$

$$L_{екв} = L_1 + L_2 \pm 2M$$

Паралелно съединение на два индуктивно свързани елемента.



фиг.12а

$$i(t)=i_1(t)+i_{2_1}(t)$$

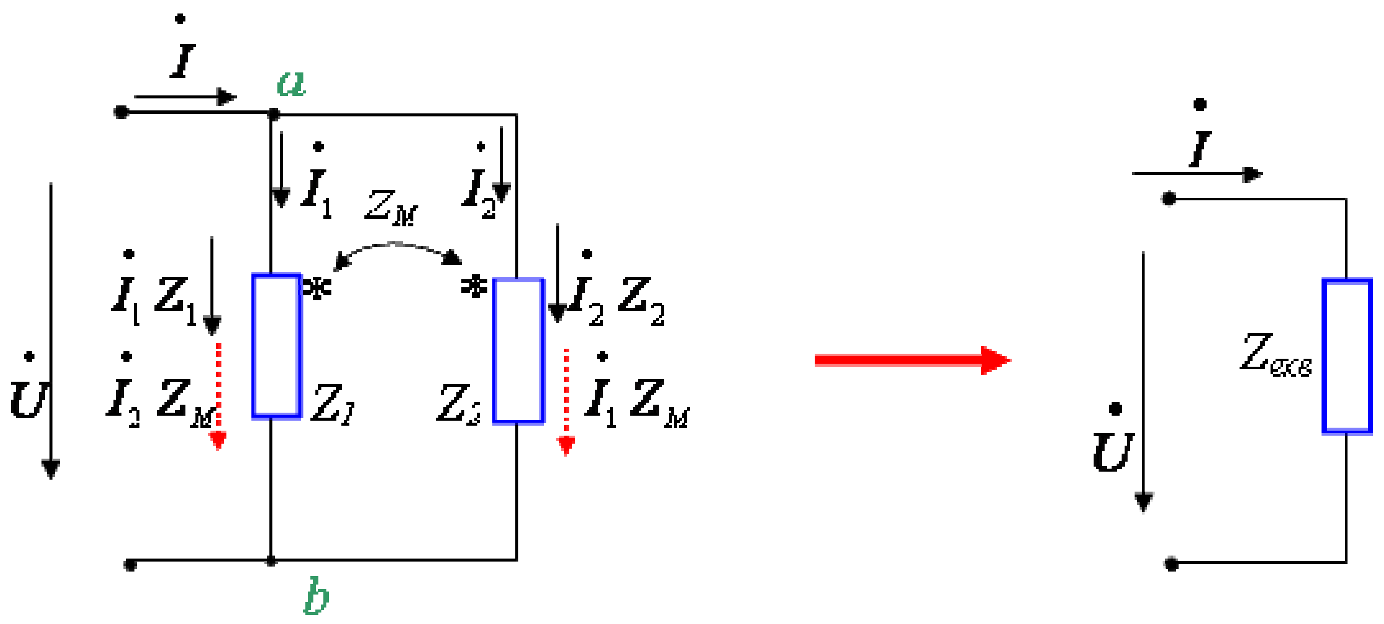
$$u(t)=u_{R_1}(t)+u_{L_1}(t)+u_{M_1}(t)$$

$$u(t)=u_{R_2}(t)+u_{L_2}(t)+u_{M_2}(t)$$

$$i(t)=i_1(t)+i_{2_1}(t)$$

$$u(t)=R_1 i_1(t)+L_1\frac{di_1}{dt}+M\frac{di_2}{dt}$$

$$u(t)=R_2 i_2(t)+L_2\frac{di_2}{dt}+M\frac{di_1}{dt}$$



$$Z_1 = R_1 + j\omega L_1; \qquad Z_2 = R_2 + j\omega L_2; \qquad Z_M = j\omega M$$

$$\dot{I} = \dot{I}_1 + \dot{I}_2$$

$$\dot{U} = \dot{I}_1 (R_1 + j\omega L_1) + \dot{I}_2 . j\omega M$$

$$\dot{U} = \dot{I}_2 (R_2 + j\omega L_2) + \dot{I}_1 . j\omega M$$

$$\dot{I} - \dot{I}_1 - \dot{I}_2 = 0$$

$$\dot{I}_1 Z_1 + \dot{I}_2 . Z_M = \dot{U}$$

$$\dot{I}_1 Z_M + \dot{I}_2 . Z_2 = \dot{U}$$

$$\dot{I} - \dot{I}_1 - \dot{I}_2 = 0$$

$$\dot{I}_1 Z_1 + \dot{I}_2 . Z_M = \dot{U}$$

$$\dot{I}_1 Z_M + \dot{I}_2 . Z_2 = \dot{U}$$

$$\dot{I}_1 = \frac{\begin{vmatrix} \dot{U} & Z_M \\ \dot{U} & Z_2 \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_M \\ Z_M & Z_2 \end{vmatrix}} = \frac{\dot{U}(Z_2 - Z_M)}{Z_1 Z_2 - Z_M{}^2}$$

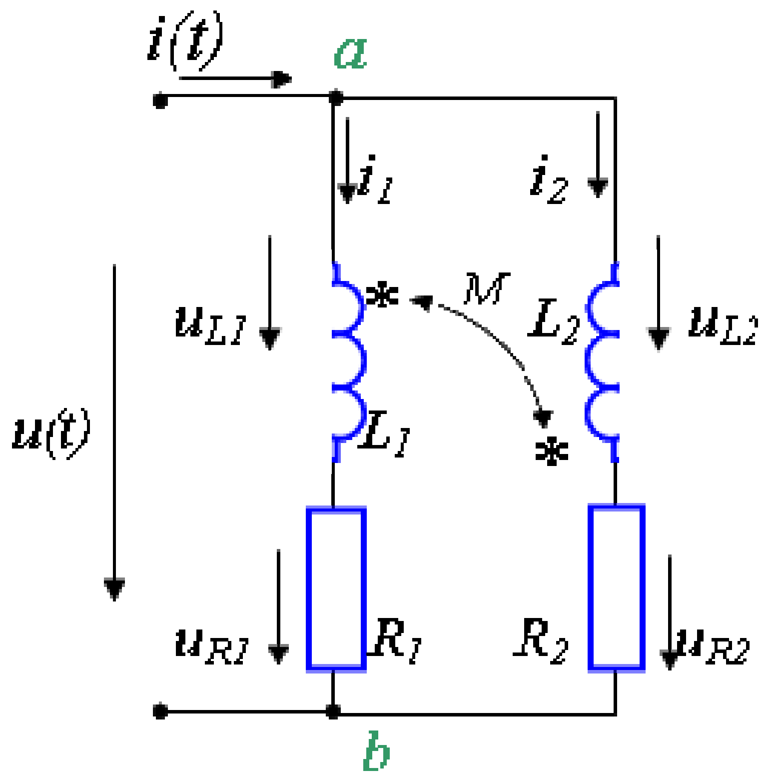
$$\dot{I}_2 = \frac{\begin{vmatrix} Z_1 & \dot{U} \\ Z_M & \dot{U} \end{vmatrix}}{\begin{vmatrix} Z_1 & Z_M \\ Z_M & Z_2 \end{vmatrix}} = \frac{\dot{U}(Z_1 - Z_M)}{Z_1 Z_2 - Z_M{}^2}$$

$$\dot{I} = \dot{I}_1 + \dot{I}_2$$

$$\dot{I} = \frac{\dot{U}(Z_1 + Z_2 - 2Z_M)}{Z_1 Z_2 - Z_M{}^2}$$

$$Z_{екв} = \frac{\dot{U}}{\dot{I}} = \frac{Z_1 Z_2 - Z_M{}^2}{Z_1 + Z_2 - 2Z_M}$$

Несъгласувано свързване



фиг.13а

$$i(t) = i_1(t) + i_{2_1}(t)$$

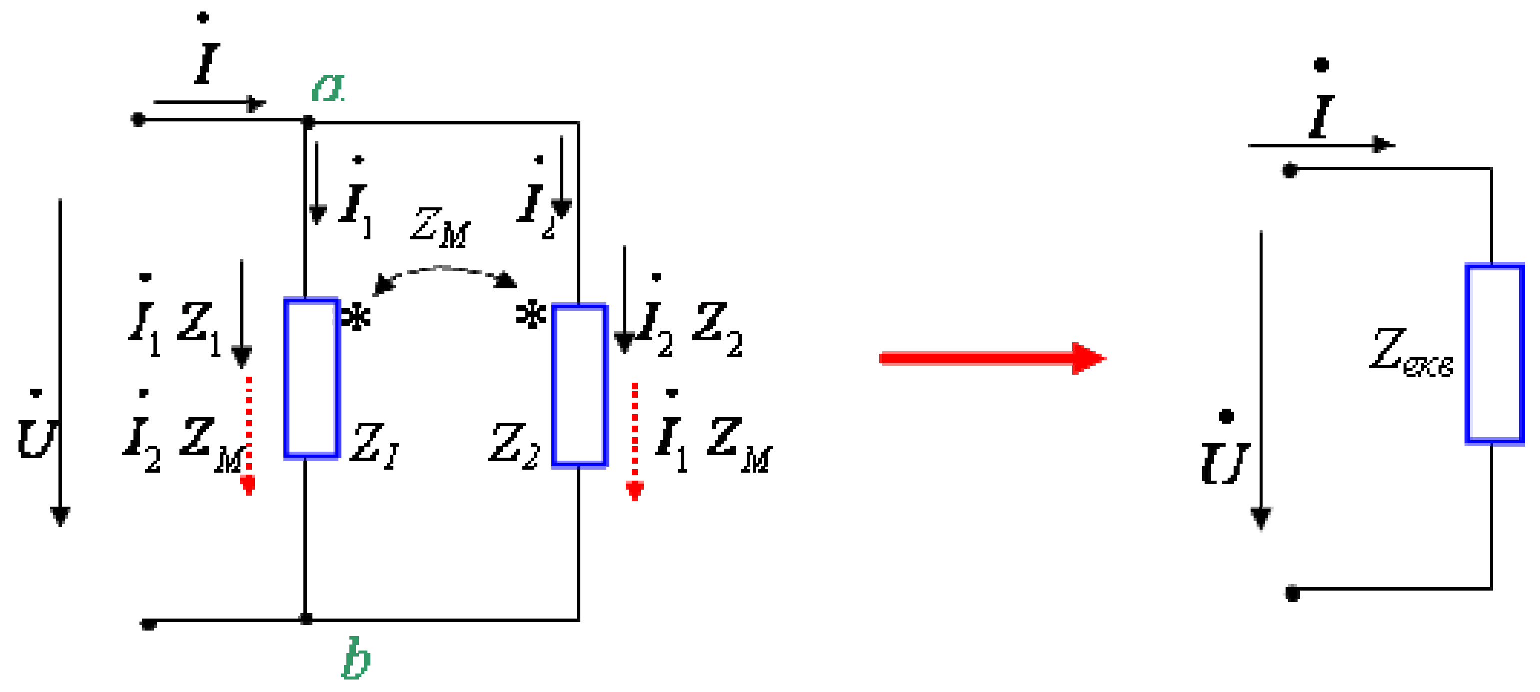
$$u(t) = u_{R_1}(t) + u_{L_1}(t) - u_{M_1}(t)$$

$$u(t) = u_{R_2}(t) + u_{L_2}(t) - u_{M_2}(t)$$

$$i(t) = i_1(t) + i_{2_1}(t)$$

$$u(t) = R_1 i_1(t) + L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$u(t) = R_2 i_2(t) + L_2 \frac{di_2}{dt} - M \frac{di_1}{dt}$$



$$\dot{I} = \dot{I}_1 + \dot{I}_2$$

$$\dot{U} = \dot{I}_1 (R_1 + j\omega L_1) - \dot{I}_2 . j\omega M$$

$$\dot{U} = \dot{I}_2 (R_2 + j\omega L_2) - \dot{I}_1 . j\omega M$$

$$\dot{I} - \dot{I}_1 - \dot{I}_2 = 0$$

$$\dot{I}_1 Z_1 - \dot{I}_2 . Z_M = \dot{U}$$

$$\dot{I}_1 Z_M - \dot{I}_2 . Z_2 = \dot{U}$$

$$\dot{I} - \dot{I}_1 - \dot{I}_2 = 0$$

$$\dot{I}_1 Z_1 - \dot{I}_2 . Z_M = \dot{U}$$

$$\dot{I}_1 Z_M - \dot{I}_2 . Z_2 = \dot{U}$$

$$\dot{I}_1 = \frac{\begin{vmatrix} \dot{U} & -Z_M \\ \dot{U} & Z_2 \end{vmatrix}}{\begin{vmatrix} Z_1 & -Z_M \\ -Z_M & Z_2 \end{vmatrix}} = \frac{\dot{U}(Z_2 + Z_M)}{Z_1 Z_2 - Z_M{}^2}$$

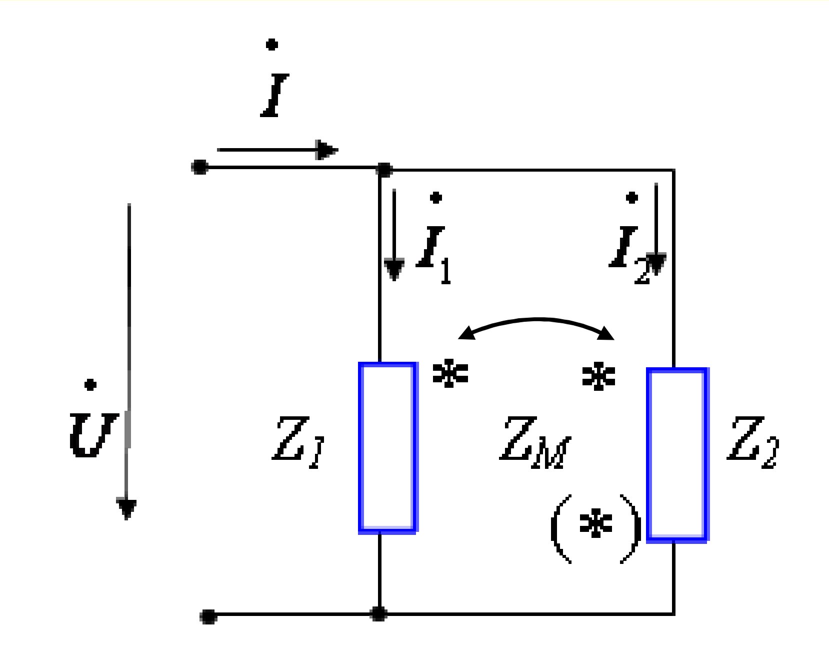
$$\dot{I}_2 = \frac{\begin{vmatrix} Z_1 & \dot{U} \\ -Z_M & \dot{U} \end{vmatrix}}{\begin{vmatrix} Z_1 & -Z_M \\ -Z_M & Z_2 \end{vmatrix}} = \frac{\dot{U}(Z_1 + Z_M)}{Z_1 Z_2 - Z_M{}^2}$$

$$\dot{I} = \dot{I}_1 + \dot{I}_2$$

$$\dot{I} = \frac{\dot{U}(Z_1 + Z_2 + 2Z_M)}{Z_1 Z_2 - Z_M{}^2}$$

$$Z_{екв} = \frac{\dot{U}}{\dot{I}} = \frac{Z_1 Z_2 - Z_M{}^2}{Z_1 + Z_2 + 2Z_M}$$

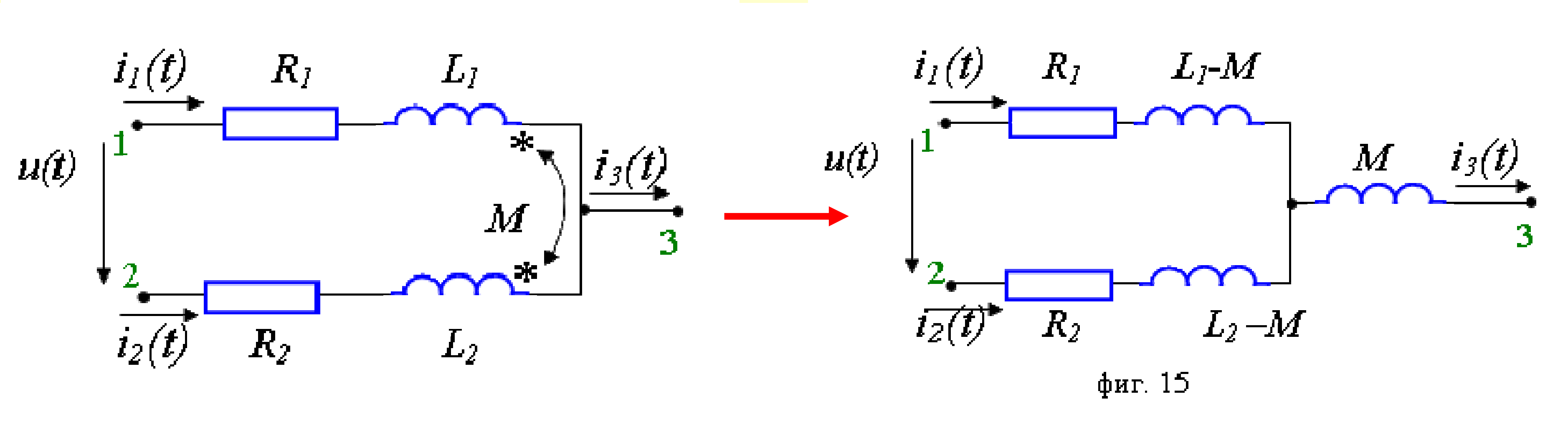
Извод



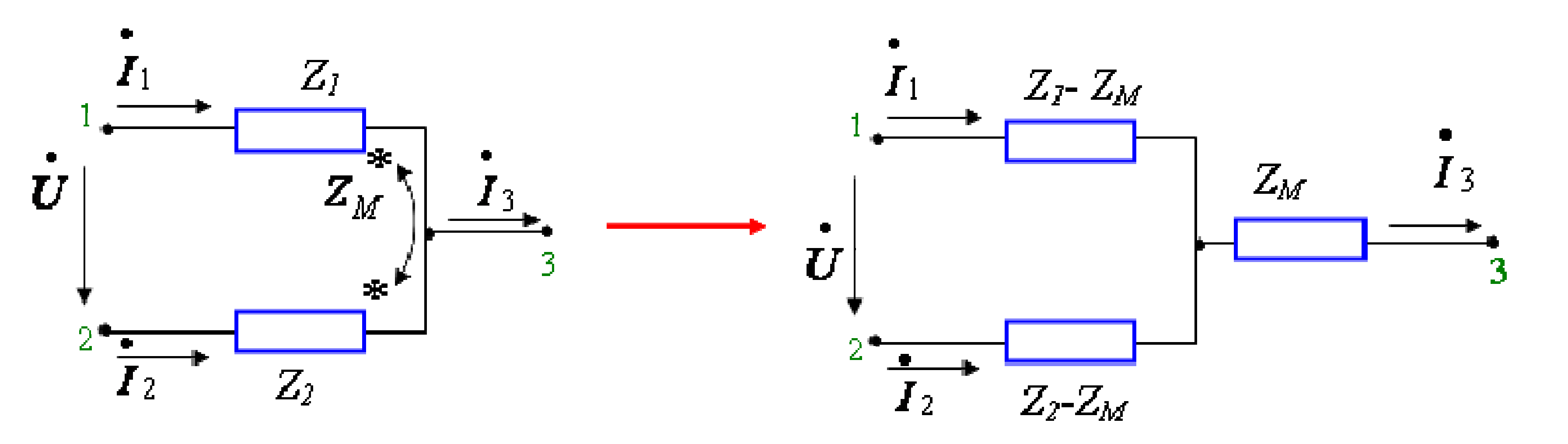
$$Z_{екв} = \frac{Z_1 Z_2 - Z_M{}^2}{Z_1 + Z_2 \mp 2Z_M}$$

Преобразуване на триполюсно съединение с индуктивна връзка

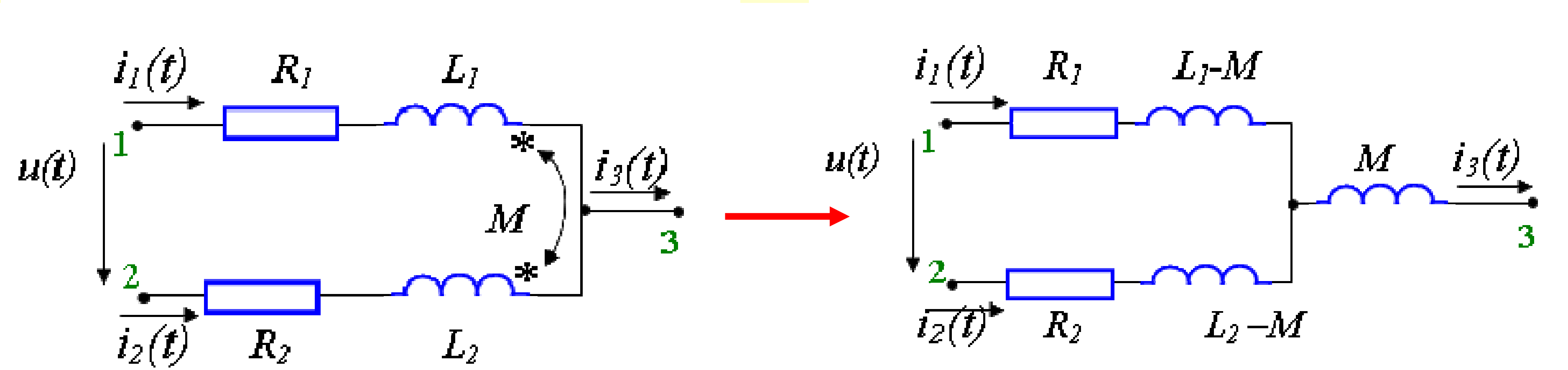
Съгласувано свързване



фиг. 15



Доказателство



$$\dot{I}_3 = \dot{I}_1 + \dot{I}_2$$

$$\dot{U}_{13} = \dot{I}_1 (R_1 + j\omega L_1) + \dot{I}_2 . j\omega M$$

$$\dot{U}_{23} = \dot{I}_2 (R_2 + j\omega L_2) + \dot{I}_1 . j\omega M$$

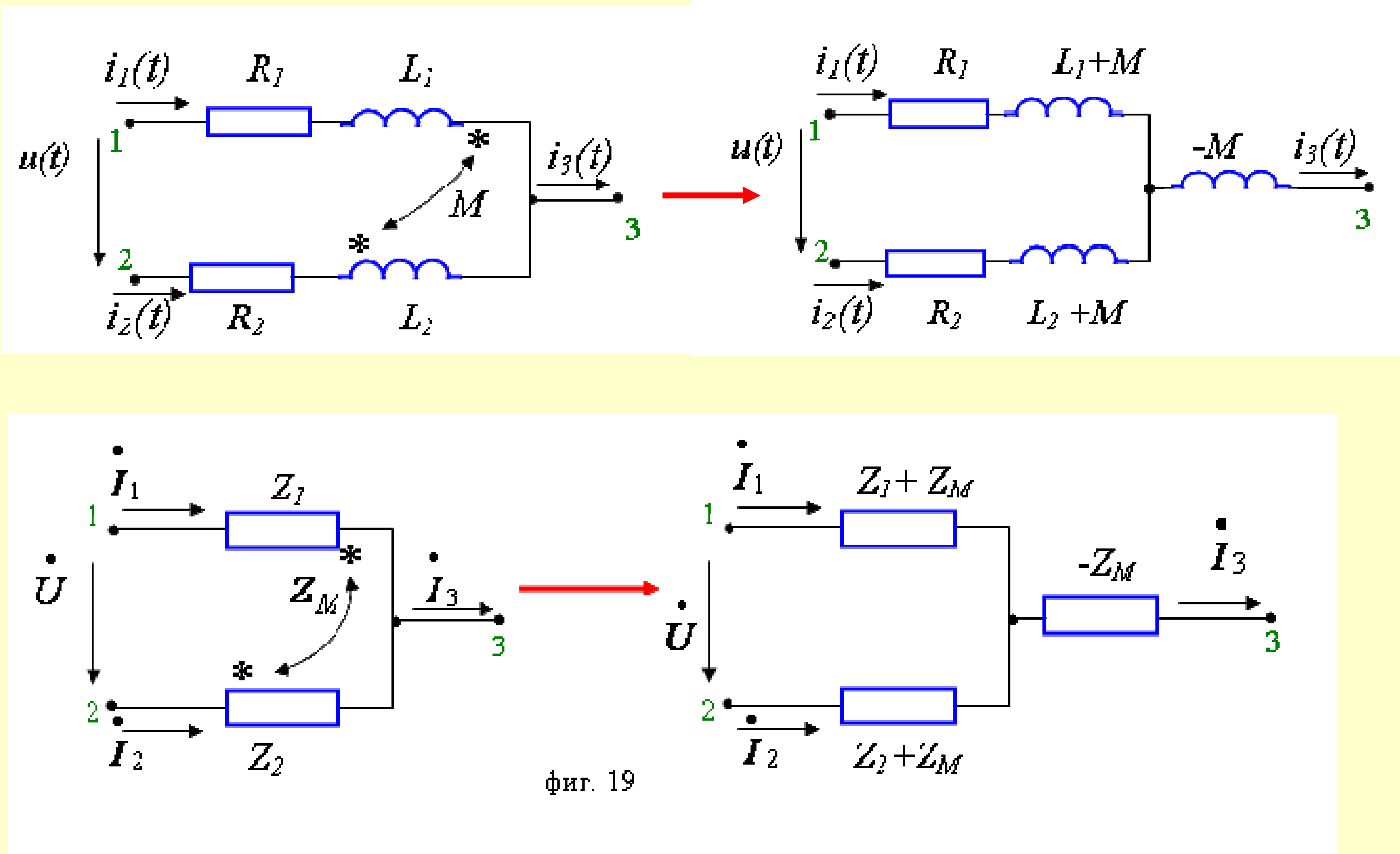
$$\dot{U}_{13} = \dot{I}_1 (R_1 + j\omega L_1) + (\dot{I}_3 - \dot{I}_1). j\omega M$$

$$\dot{U}_{23} = \dot{I}_2 (R_2 + j\omega L_2) + (\dot{I}_3 - \dot{I}_2). j\omega M$$

$$\dot{U}_{13} = \dot{I}_1 R_1 + I_1 j\omega (L_1 - M) + \dot{I}_3 . j\omega M$$

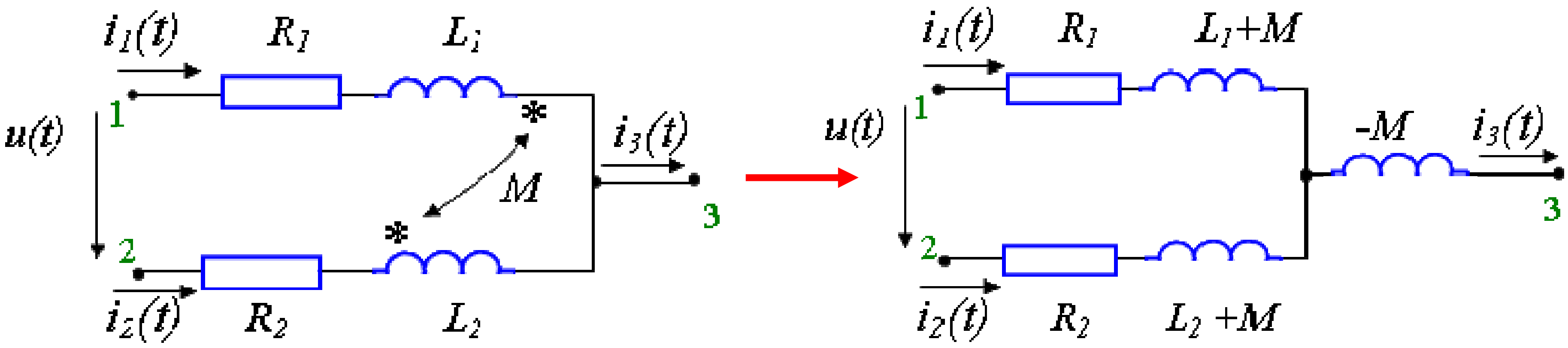
$$\dot{U}_{23} = \dot{I}_2 R_2 + \dot{I}_2 j\omega (L_2 - M) + \dot{I}_3 . j\omega M$$

Несъгласувано свързване



фиг. 19

<u>Доказателство</u>



$$\dot{I}_3 = \dot{I}_1 + \dot{I}_2$$

$$\dot{U}_{13} = \dot{I}_1(R_1 + j\omega L_1) - \dot{I}_2 . j\omega M$$

$$\dot{U}_{23} = \dot{I}_2(R_2 + j\omega L_2) - \dot{I}_1 . j\omega M$$

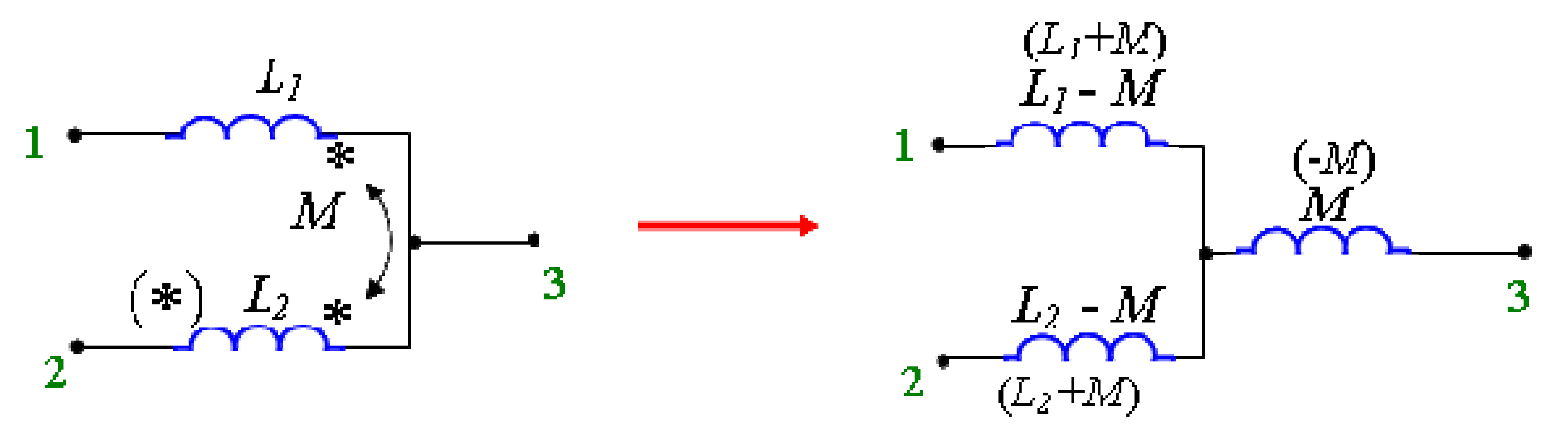
$$\dot{U}_{13} = \dot{I}_1(R_1 + j\omega L_1) - (\dot{I}_3 - \dot{I}_1). j\omega M$$

$$\dot{U}_{23} = \dot{I}_2(R_2 + j\omega L_2) - (\dot{I}_3 - \dot{I}_2). j\omega M$$

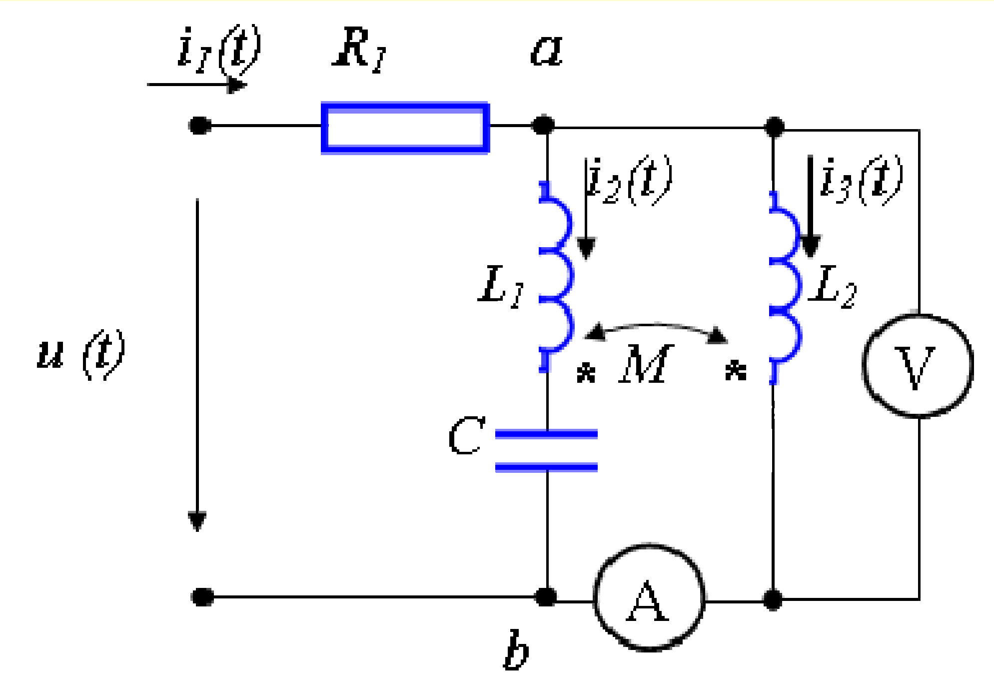
$$\dot{U}_{13} = \dot{I}_1 R_1 + I_1 j\omega(L_1 + M) - \dot{I}_3 . j\omega M$$

$$\dot{U}_{23} = \dot{I}_2 R_2 + \dot{I}_2 j\omega(L_2 + M) - \dot{I}_3 . j\omega M$$

Извод:

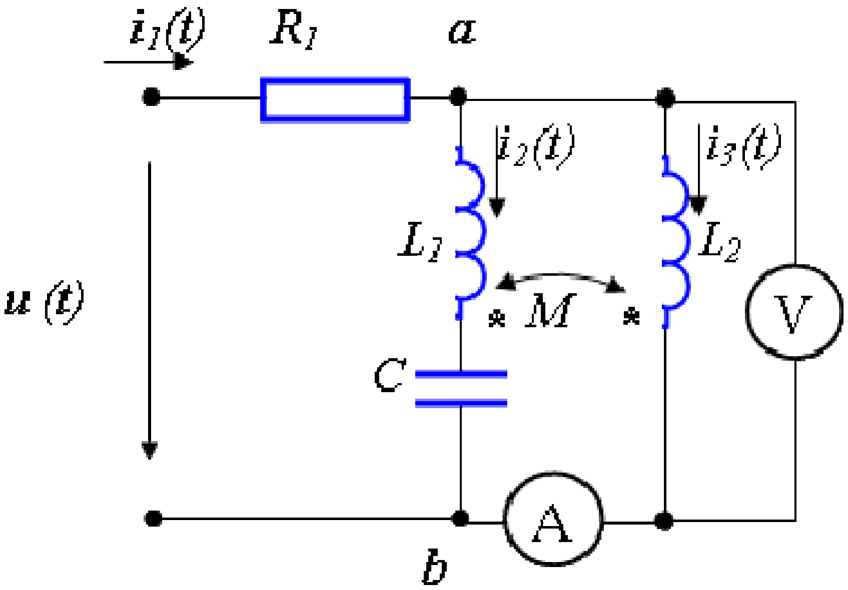


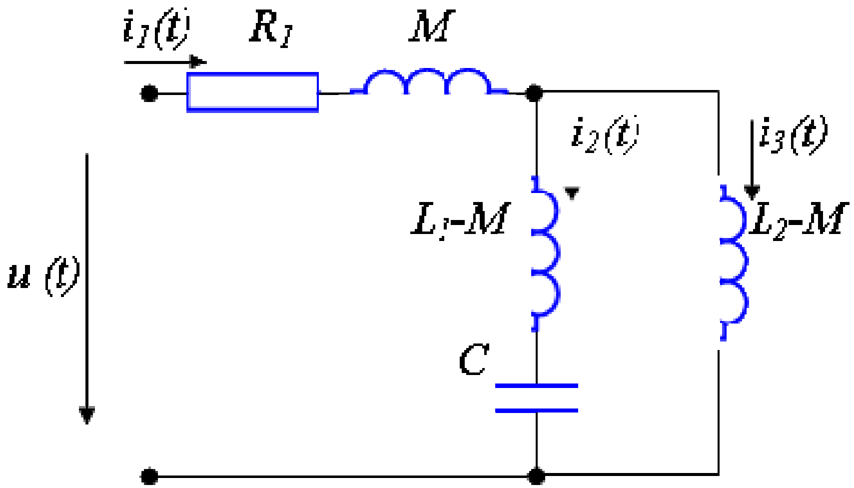
Пример за анализ на верига с индуктивни връзки посредством отстраняване на индуктивната връзка:

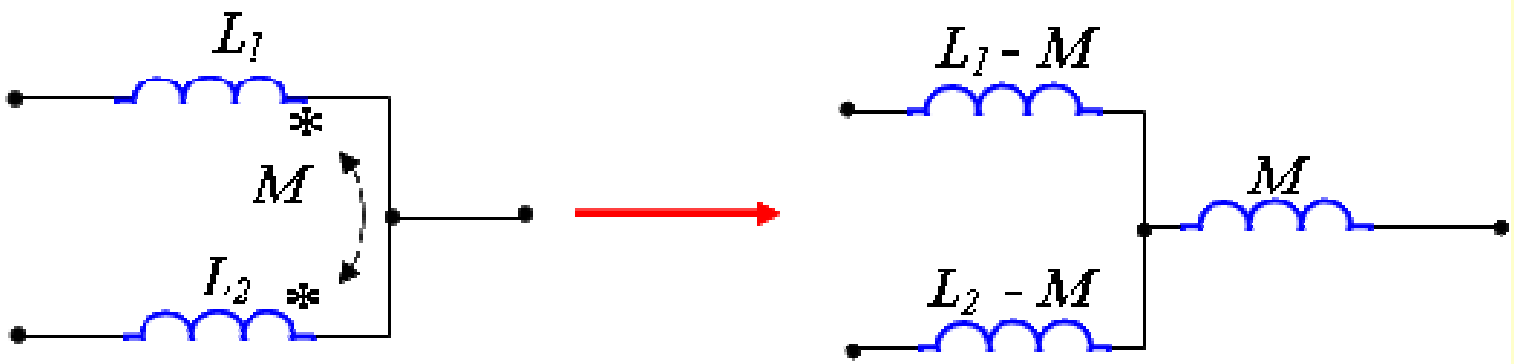


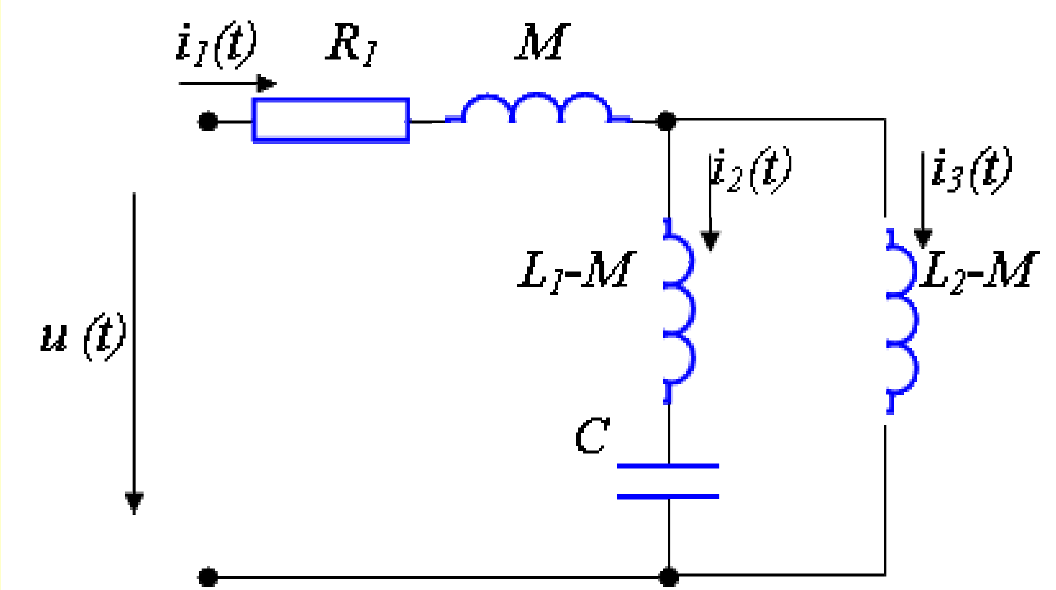
$u(t) = 141\sin(\omega t+90) V$

$f=160 Hz$,
$L_1=40\ mH$,
$L_2=30\ mH$,
$M=10\ mH$,
$C=100\ \mu F$
$R_1 = 10\Omega$,

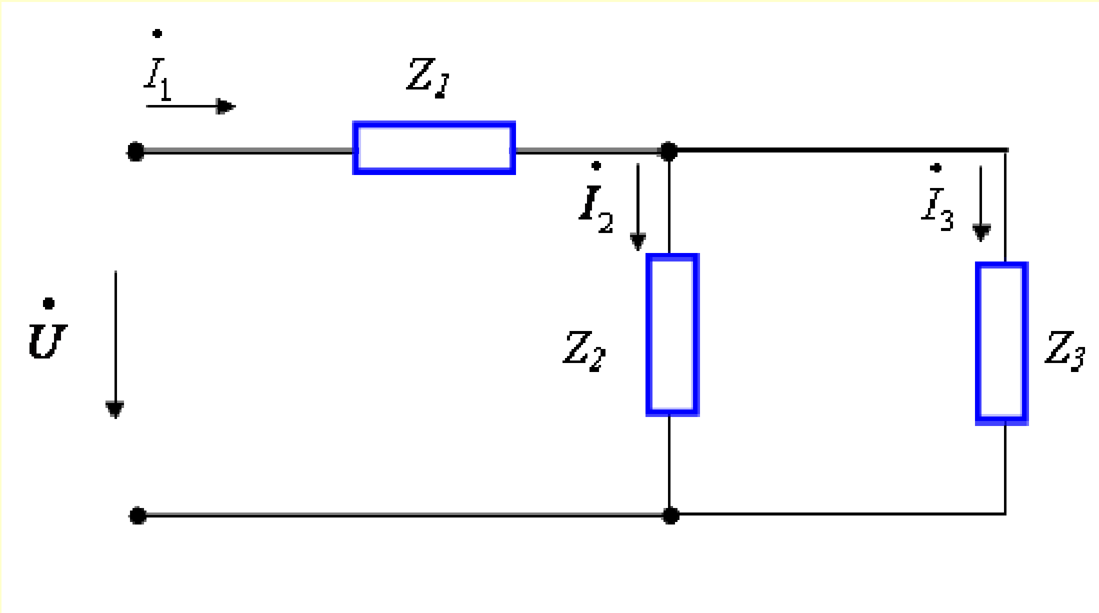

i1(t)
R1
a
i2(t)
i3(t)
L1
L2
M
u (t)
C
V
A
b


i1(t)
R1
M
i2(t)
i3(t)
L1-M
L2-M
u (t)
C

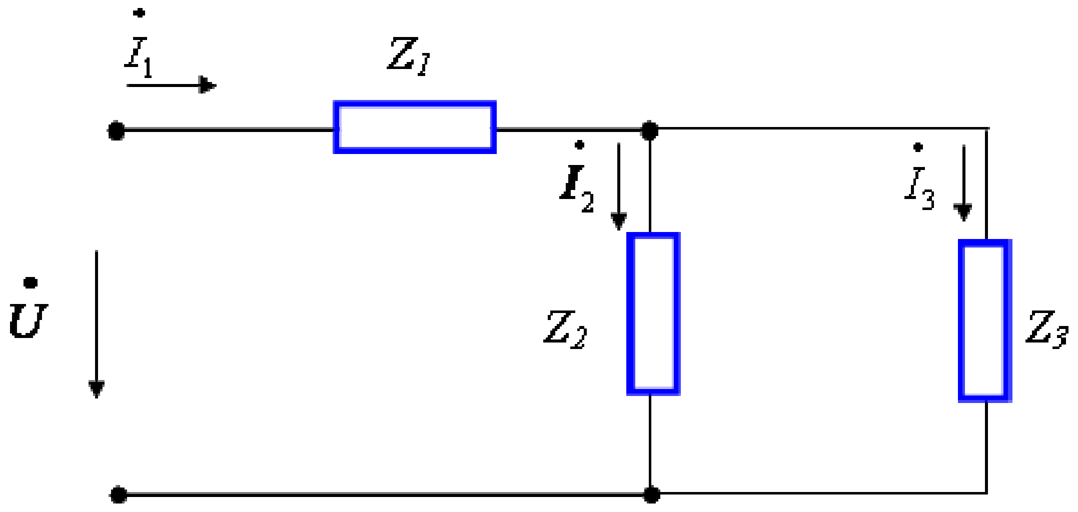

L1
M
L2
L1 - M
L2 - M
M



i1(t)
R1
M
i2(t)
i3(t)
L1-M
L2-M
u (t)
C



İ1
Z1
İ2
İ3
U̇
Z2
Z3



$u(t) = 141\sin(\omega t+90)V$

$f=160Hz,$
$L_1=40\ mH,$
$L_2=30\ mH,$
$M=10\ mH,$
$C=100\ \mu F$
$R_1 = 10\Omega,$

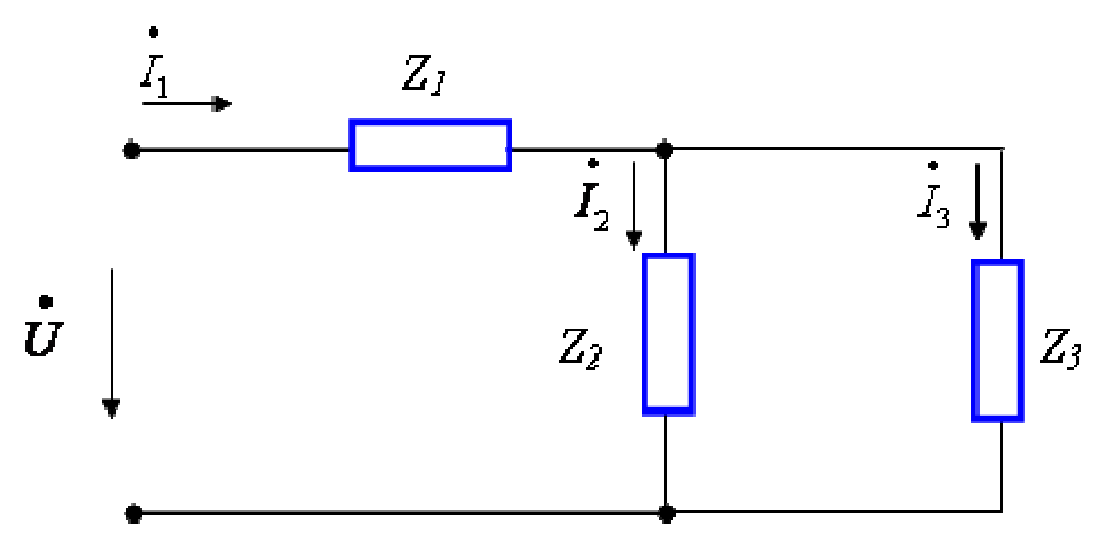
$$\dot{U} = Ue^{j\psi_u} = \frac{u_m}{\sqrt{2}}e^{j\psi_u} = \frac{141}{\sqrt{2}}e^{j90} =$$
$$100.[\cos(90) + j\sin(90)] = 100.(0+j) = j100V$$

$$\omega = 2\pi f = 2\pi.160 \approx 1000 = 10^3 rad/s$$

$$Z_1 = R_1 + j\omega M = (10 + j.10^3 10.10^{-3}) = (10 + j10)\,\Omega$$

$$Z_2 = j\omega(L_1 - M) - j\frac{1}{\omega C} = j.10^3.(40-10).10^{-3} - j\frac{1}{10^3.100.10^{-6}} = j30 - j10 = j20\Omega$$

$$Z_3 = j\omega(L_2 - M) = j.10^3(30-10).10^{-3} = j20\Omega$$

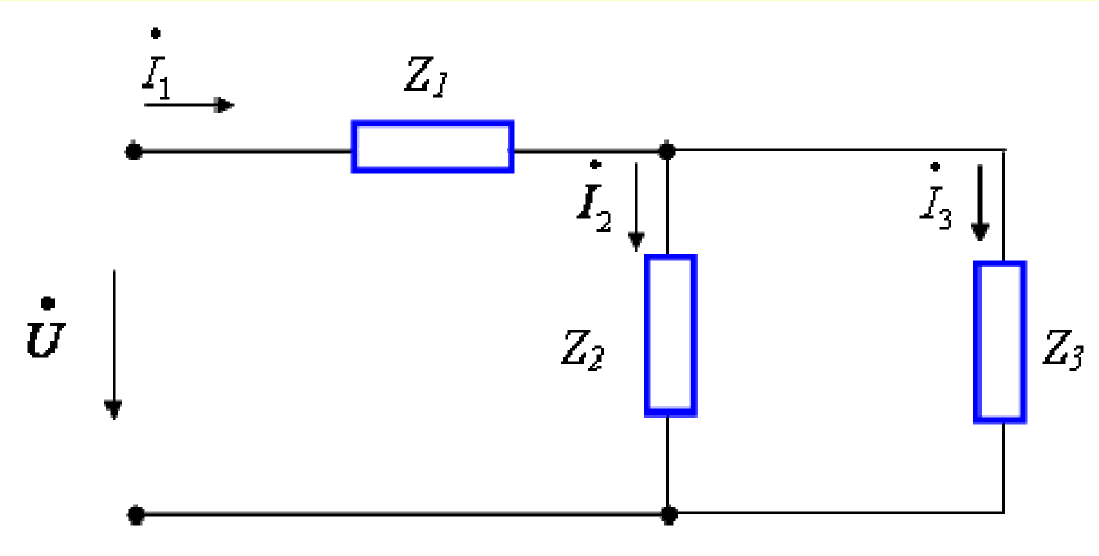


$\dot{U} = j100V$

$Z_1 = (10 + j10)\ \Omega$

$Z_2 = j20\ \Omega$

$Z_3 = j20\ \Omega$



$\dot{U} = j100V$

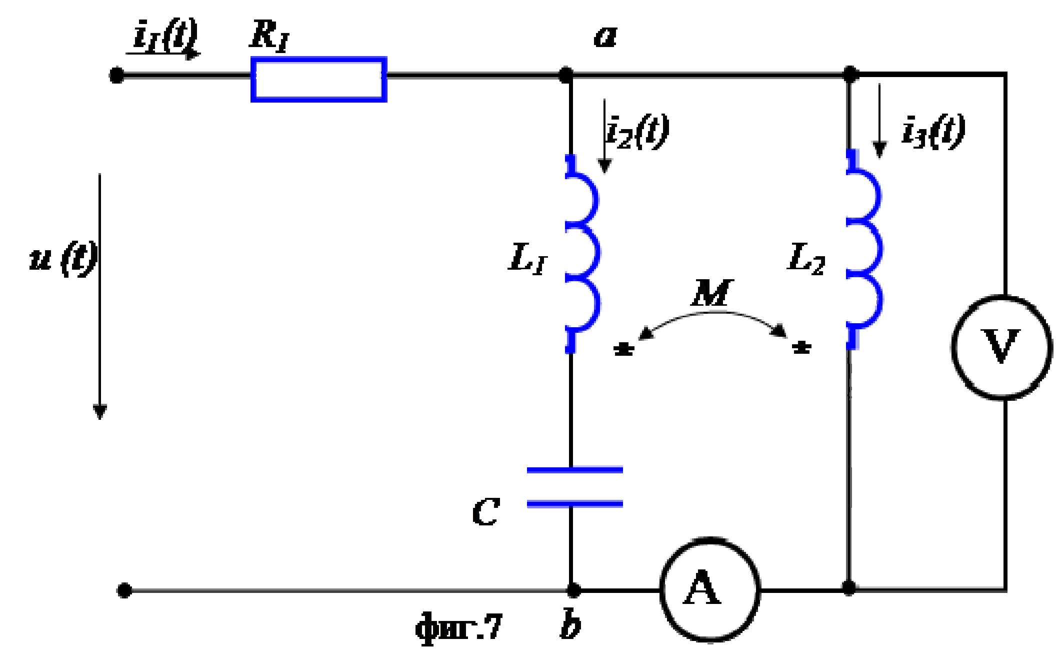
$Z_1 = (10 + j10)\,\Omega$

$Z_2 = j20\,\Omega$

$Z_3 = j20\,\Omega$

$$\dot{I}_1 = (4 + 2j)A$$

Показания на уредите:



$$\dot{I}_1 = (4 + j2) = 4{,}47e^{j26{,}56}\,A$$

$$I_A = I_3 = 2{,}24\,A$$

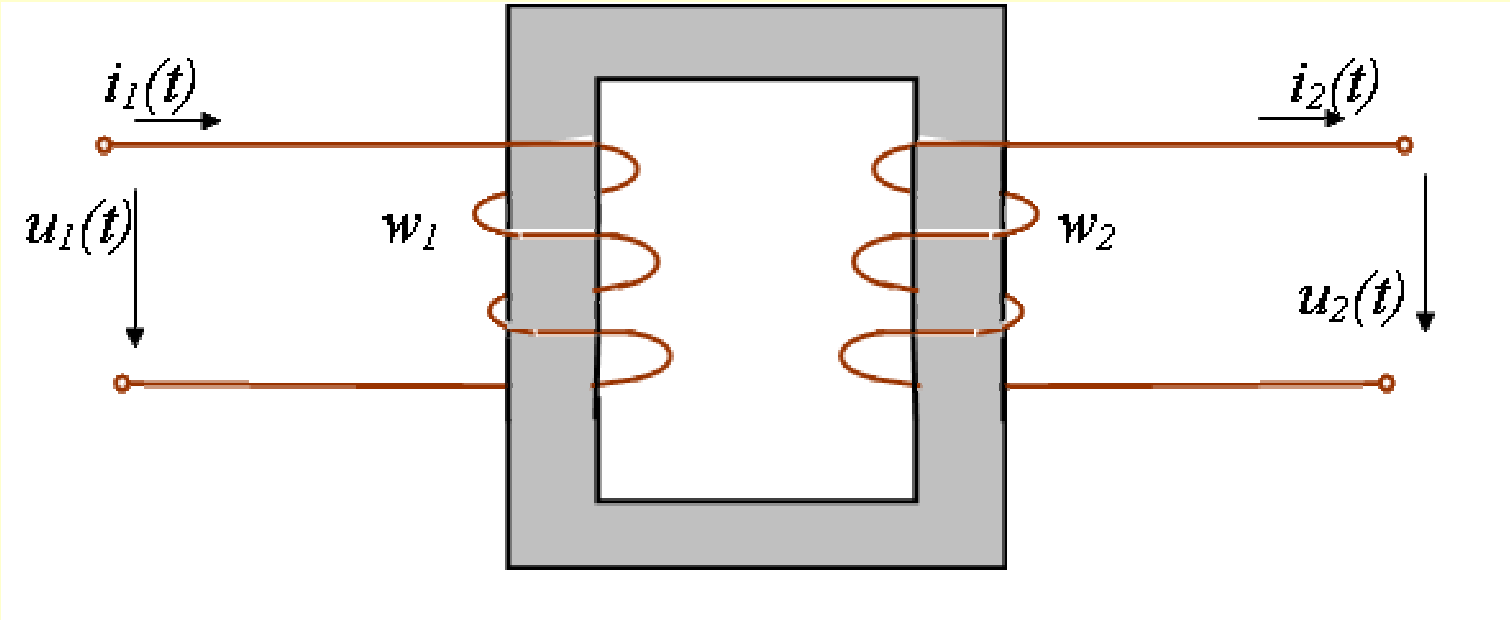
$$U_V = U_{ab}$$

$$\dot{U}_{ab} = Z_{L_2}.\dot{I}_2 + Z_M.\dot{I}_3 =$$

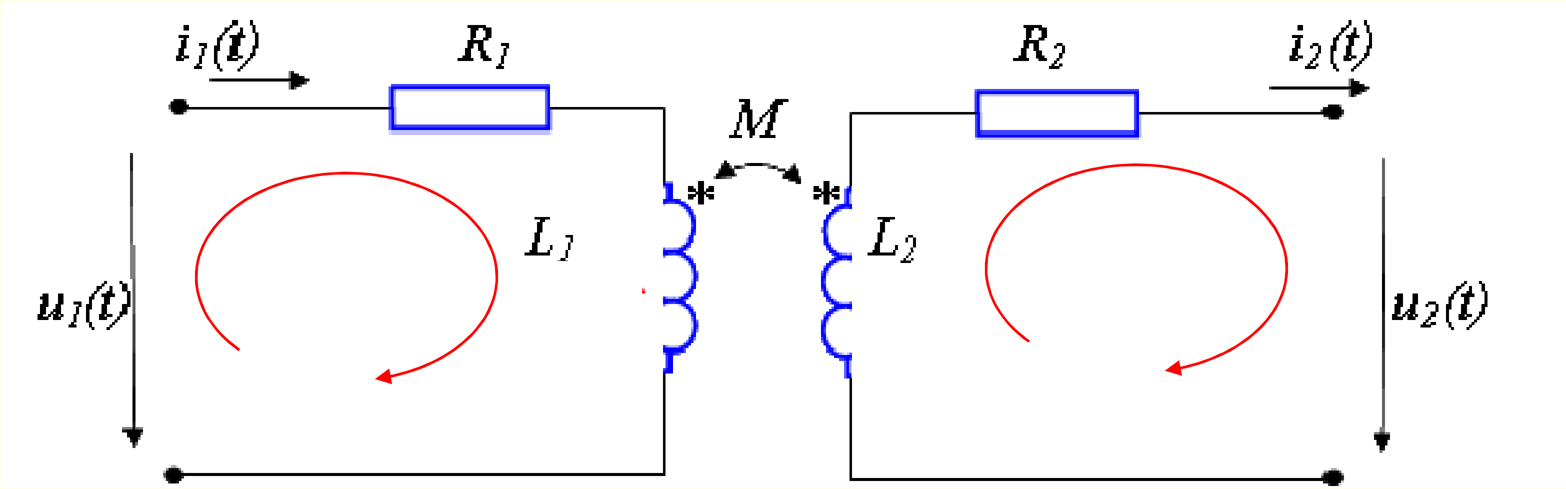
$$j\omega L_2.\dot{I}_2 + j\omega M.\dot{I}_3 = (2+j)j30 + (2+j)j10 = (-40 + j80)V$$

$$\Rightarrow U_V = U_{ab} = \sqrt{(-40)^2 + 80^2} = 40\sqrt{5} = 89{,}44V$$

Трансформаторно съединение



Уравнения на линеен трансформатор



$$u_1(t) = u_{R_1}(t) + u_{L_1}(t) - u_{M_1}(t)$$

$$-u_2(t) = u_{R_2}(t) + u_{L_2}(t) - u_{M_2}(t)$$