Решаване на примерни линейни оптимизационни задачи по симплекс метода

Организация на производство на мебели

Малка фирма разполага с **12.5** единици дървен материал, за да започне производството на маси и столове.

За **1** маса трябват **2** единици дървен материал, а за **1** стол – **1** единица.

Дистрибуторът предлага по **20 лв** на маса и **15 лв** за стол, но не иска повече от **8** стола и иска поне **2** пъти повече столове от маси.

Колко маси и стола да произведе фирмата, за да си осигури максимална печалба?

1.1 Математически модел

***Елементи на решението (управляеми променливи)*:** x_1 - брой маси

x_2 - брой столове

Целева функция – печалбата от продажбата на масите и столовете:

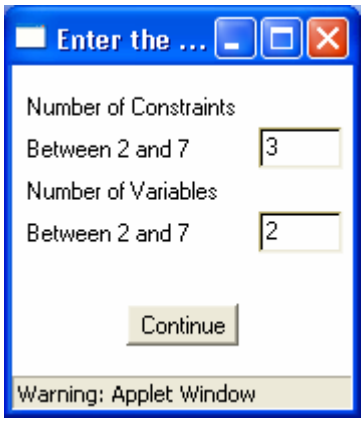
$L = 20x_1 + 15x_2 \Rightarrow$ Да се максимизира!

Ограничения:

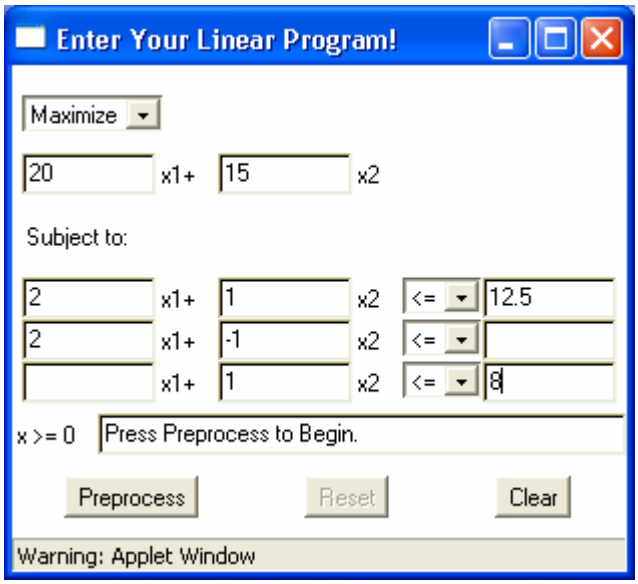
- от наличния материал: $2x_1 + x_2 \leq 12.5$
- от условията на търговеца: $x_1 \leq 8$, $x_2 \geq 2x_1$
- съображения от здрав разум : $x_1 \geq 0$, $x_2 \geq 0$

1.2 Решение

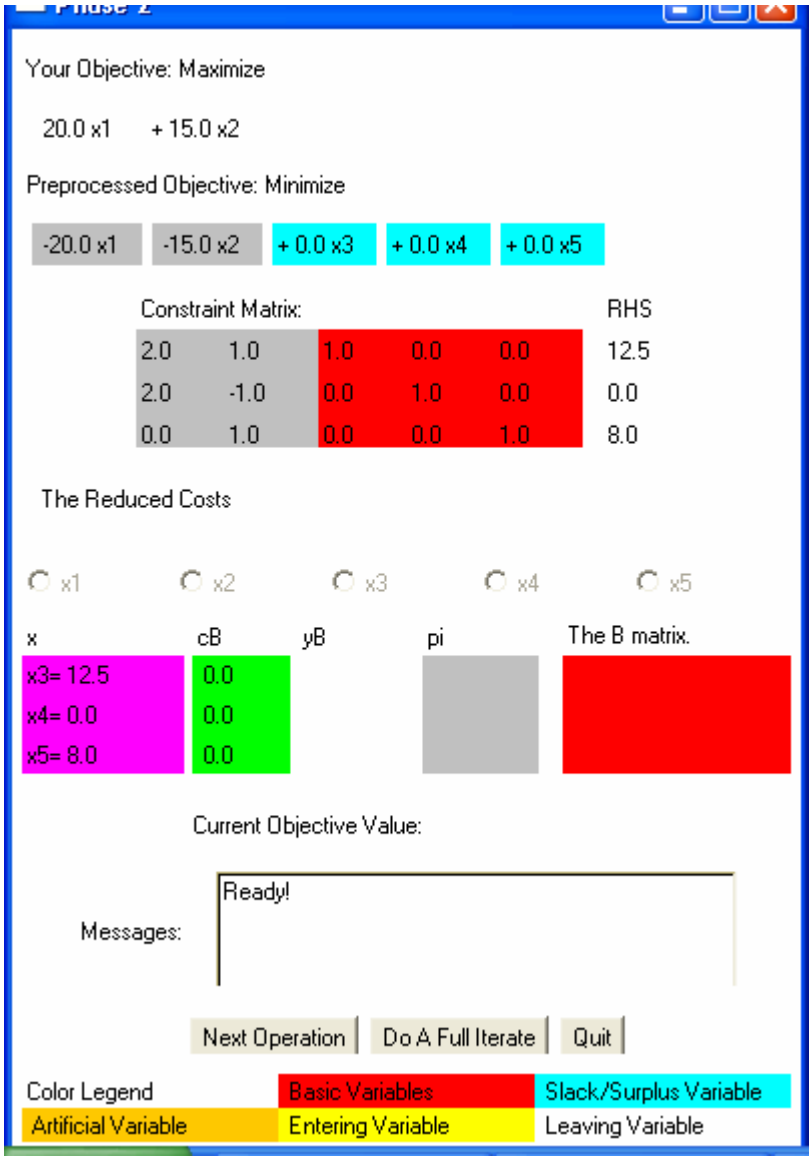
Въвеждане на броя на ограниченията и на броя на управляемите променливи

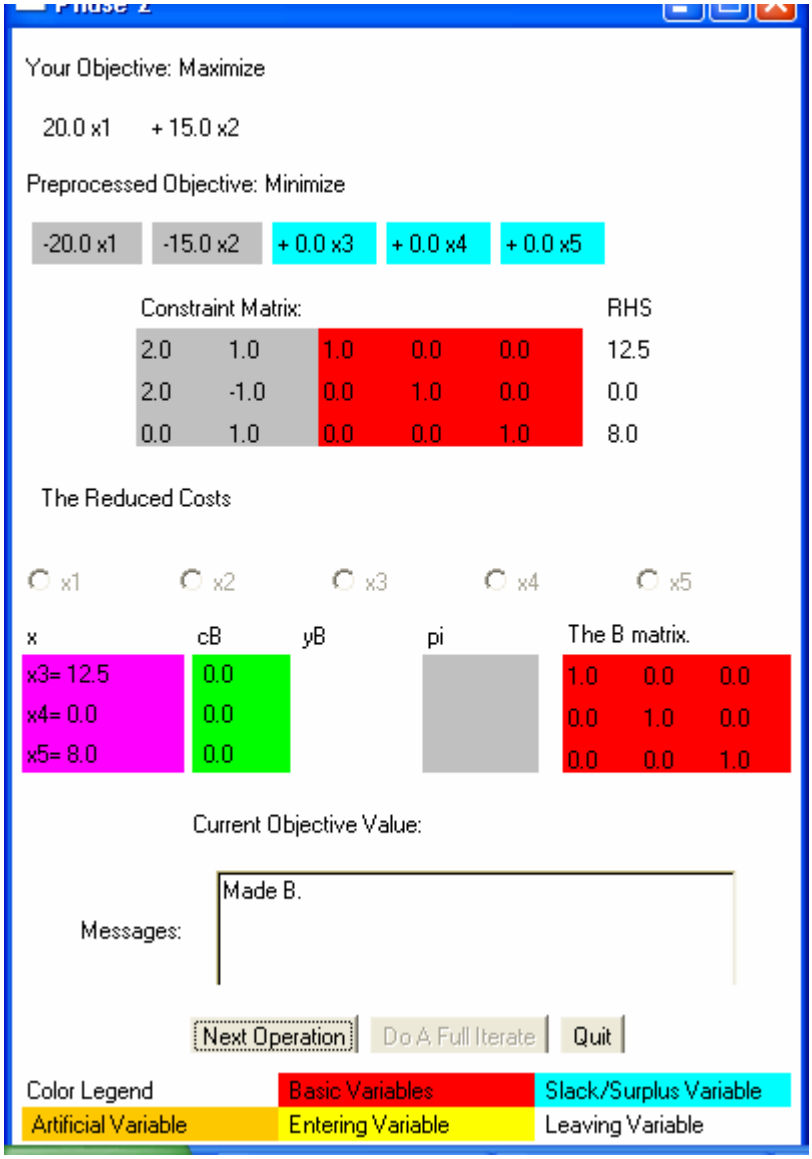


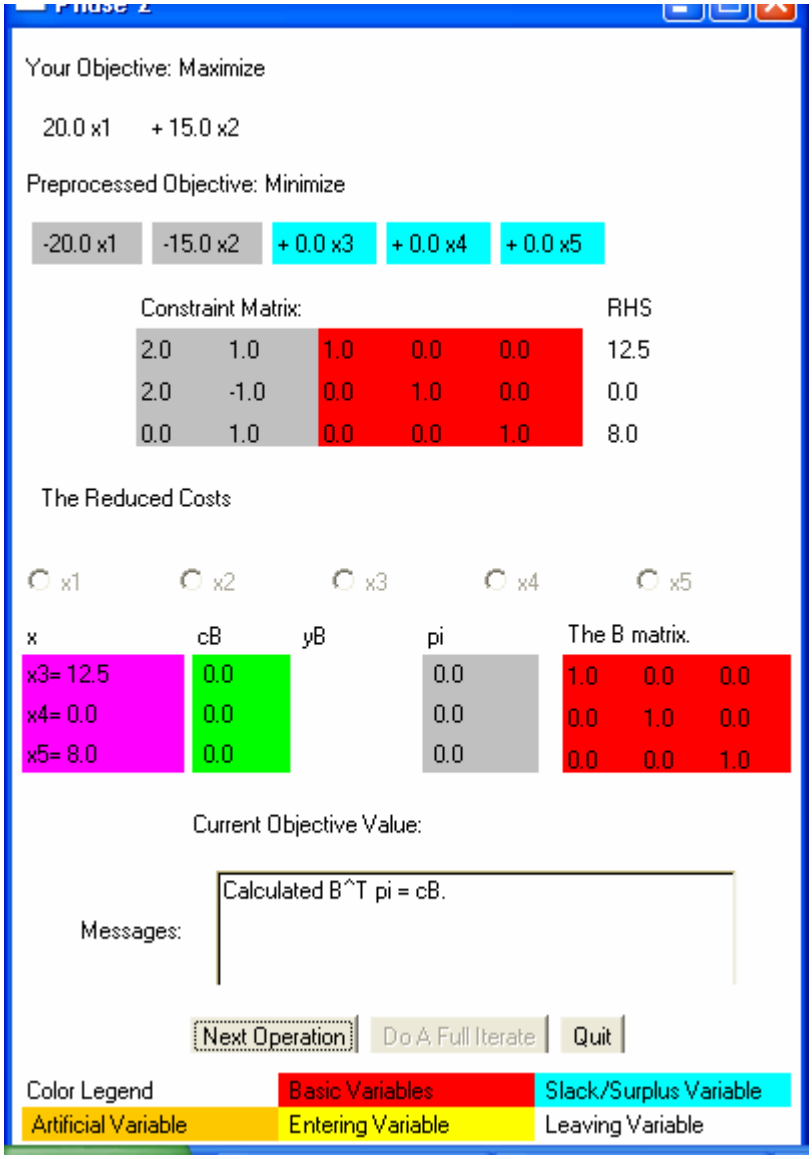
Въвеждане на ограниченията и на целевата функция

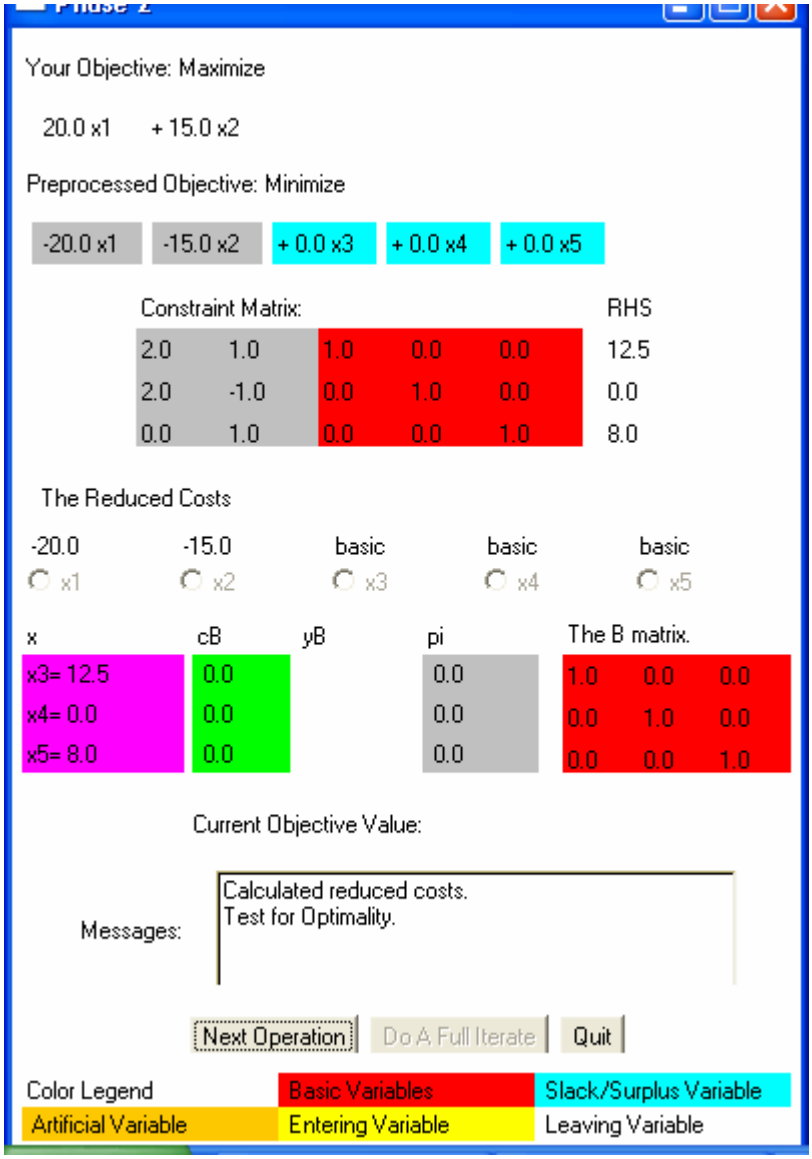


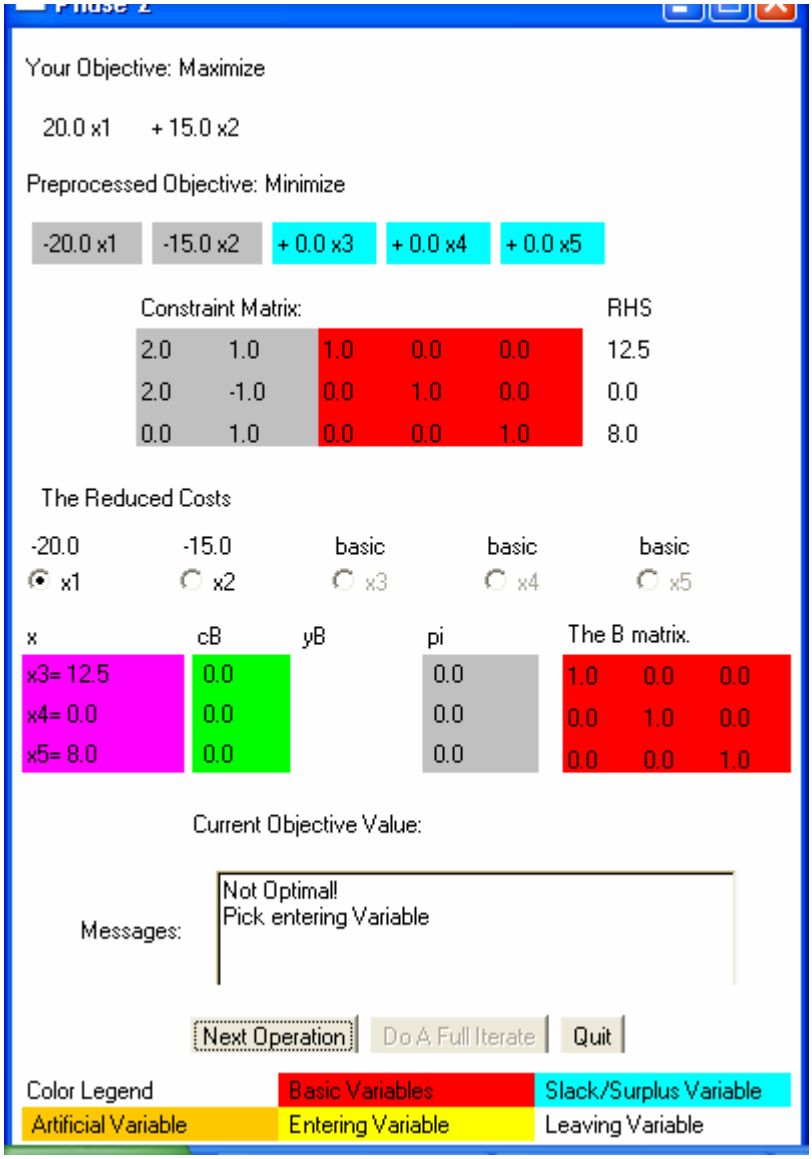
Проследяване на решението стъпка по стъпка за първата итерация:

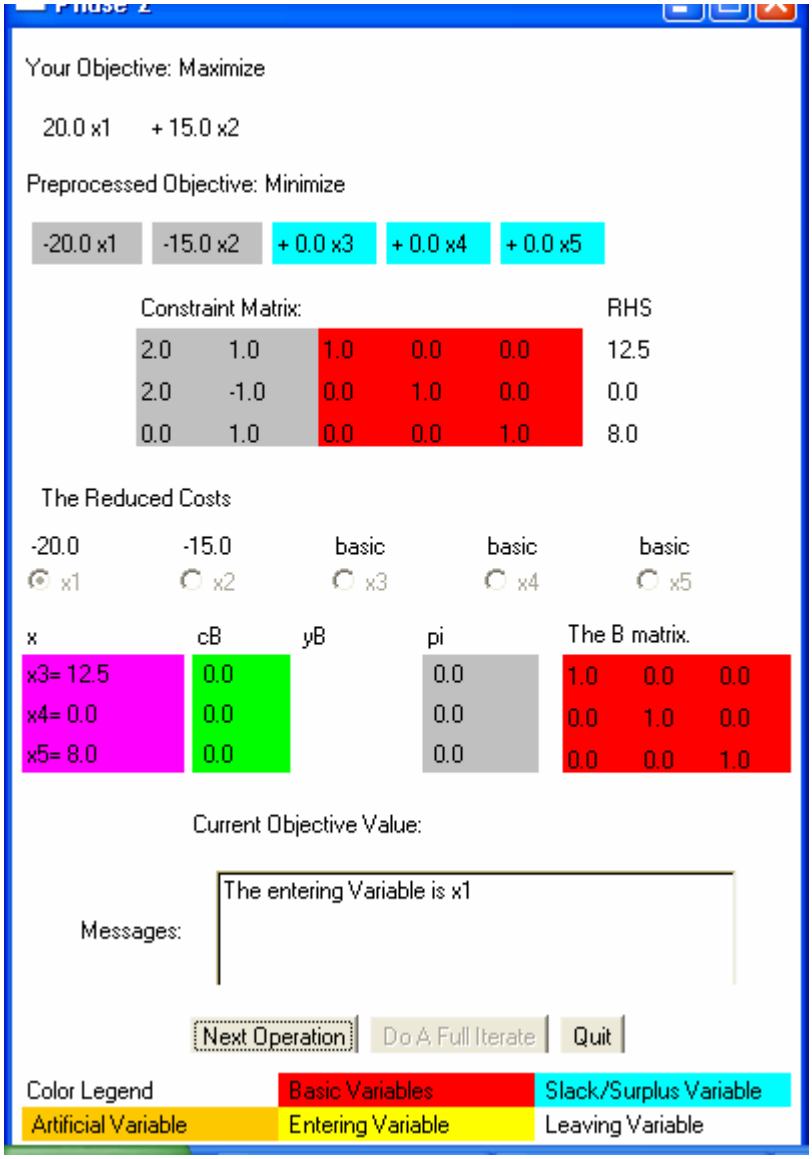


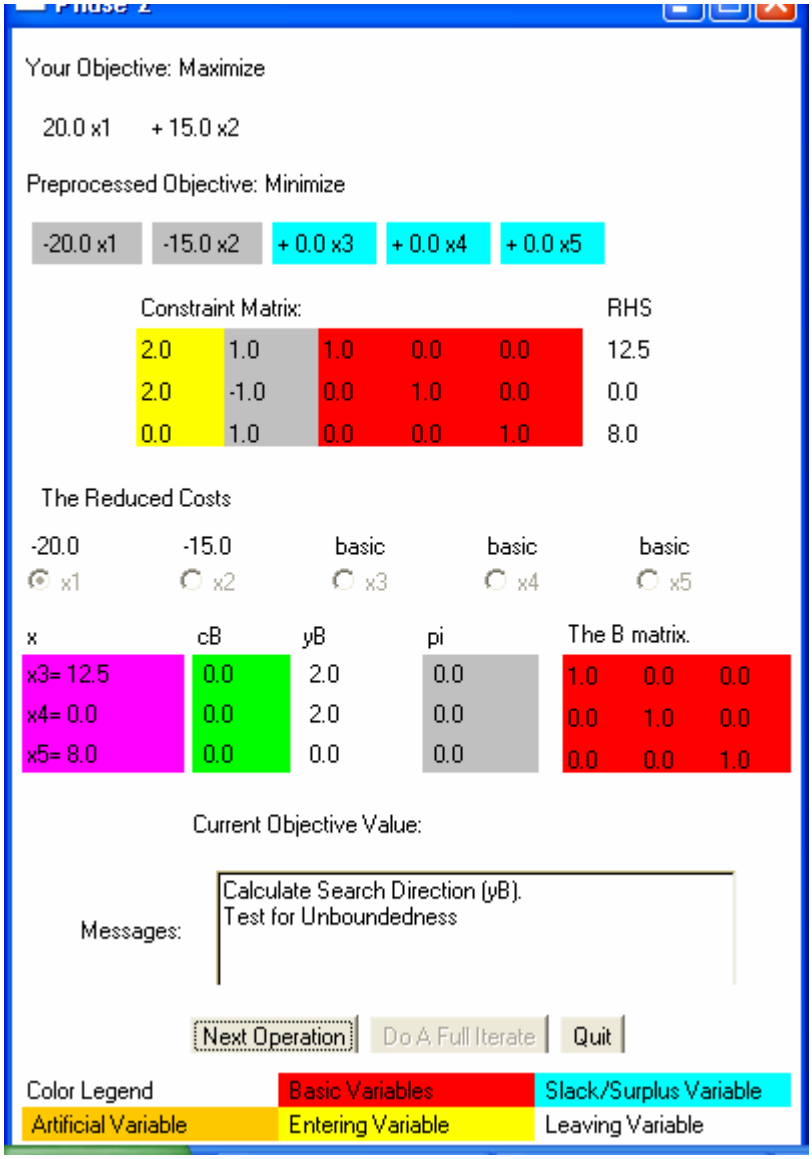

Phase 2
Your Objective: Maximize
20.0 x1 + 15.0 x2
Preprocessed Objective: Minimize
-20.0 x1
-15.0 x2
+ 0.0 x3
+ 0.0 x4
+ 0.0 x5
Constraint Matrix:
RHS
2.0 1.0 1.0 0.0 0.0 12.5
2.0 -1.0 0.0 1.0 0.0 0.0
0.0 1.0 0.0 0.0 1.0 8.0
The Reduced Costs
x1
x2
x3
x4
x5
x
cB
yB
pi
The B matrix.
x3= 12.5
x4= 0.0
x5= 8.0
0.0
0.0
0.0
1.0 0.0 0.0
0.0 1.0 0.0
0.0 0.0 1.0
Current Objective Value:
Messages:
Made B.
Next Operation
Do A Full Iterate
Quit
Color Legend
Basic Variables
Slack/Surplus Variable
Artificial Variable
Entering Variable
Leaving Variable

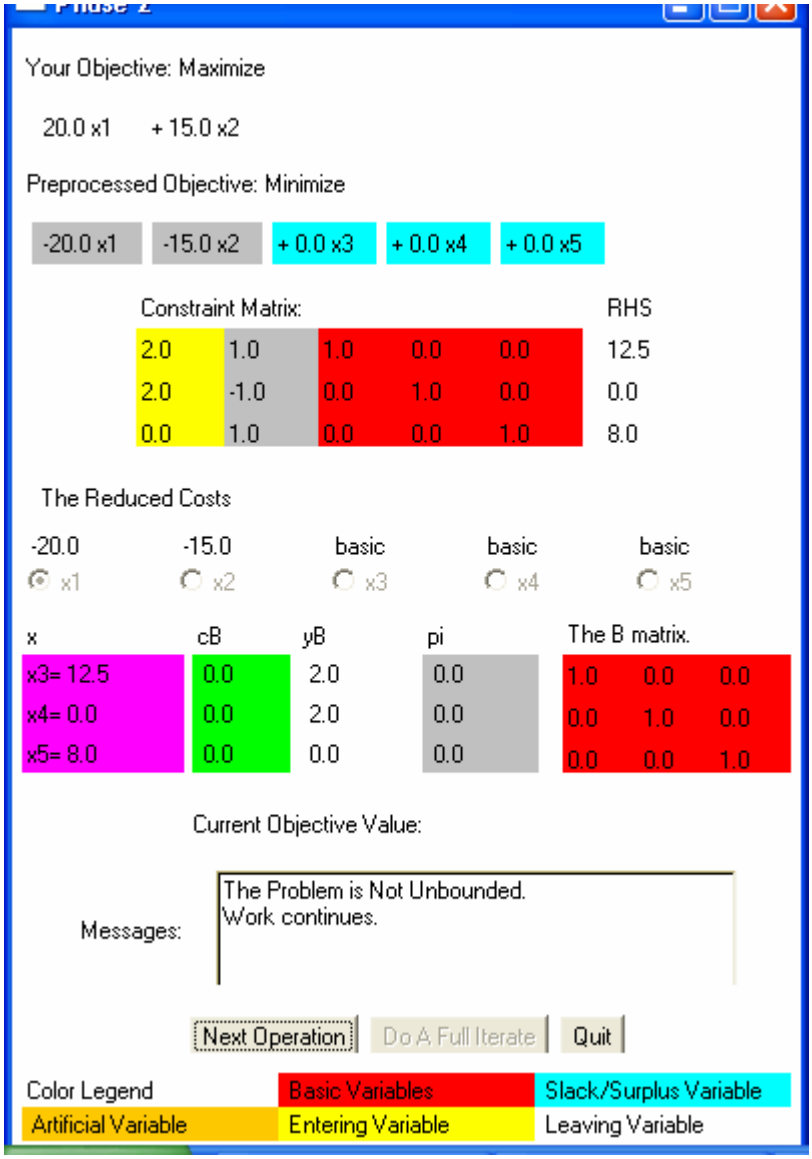

Phase 2
Your Objective: Maximize
20.0 x1 + 15.0 x2
Preprocessed Objective: Minimize
-20.0 x1
-15.0 x2
+ 0.0 x3
+ 0.0 x4
+ 0.0 x5
Constraint Matrix:
RHS
2.0 1.0 1.0 0.0 0.0 12.5
2.0 -1.0 0.0 1.0 0.0 0.0
0.0 1.0 0.0 0.0 1.0 8.0
The Reduced Costs
x1 x2 x3 x4 x5
x
cB
yB
pi
The B matrix.
x3= 12.5
x4= 0.0
x5= 8.0
0.0
0.0
0.0
0.0
0.0
0.0
1.0 0.0 0.0
0.0 1.0 0.0
0.0 0.0 1.0
Current Objective Value:
Messages:
Calculated B^T pi = cB.
Next Operation
Do A Full Iterate
Quit
Color Legend
Basic Variables
Slack/Surplus Variable
Artificial Variable
Entering Variable
Leaving Variable

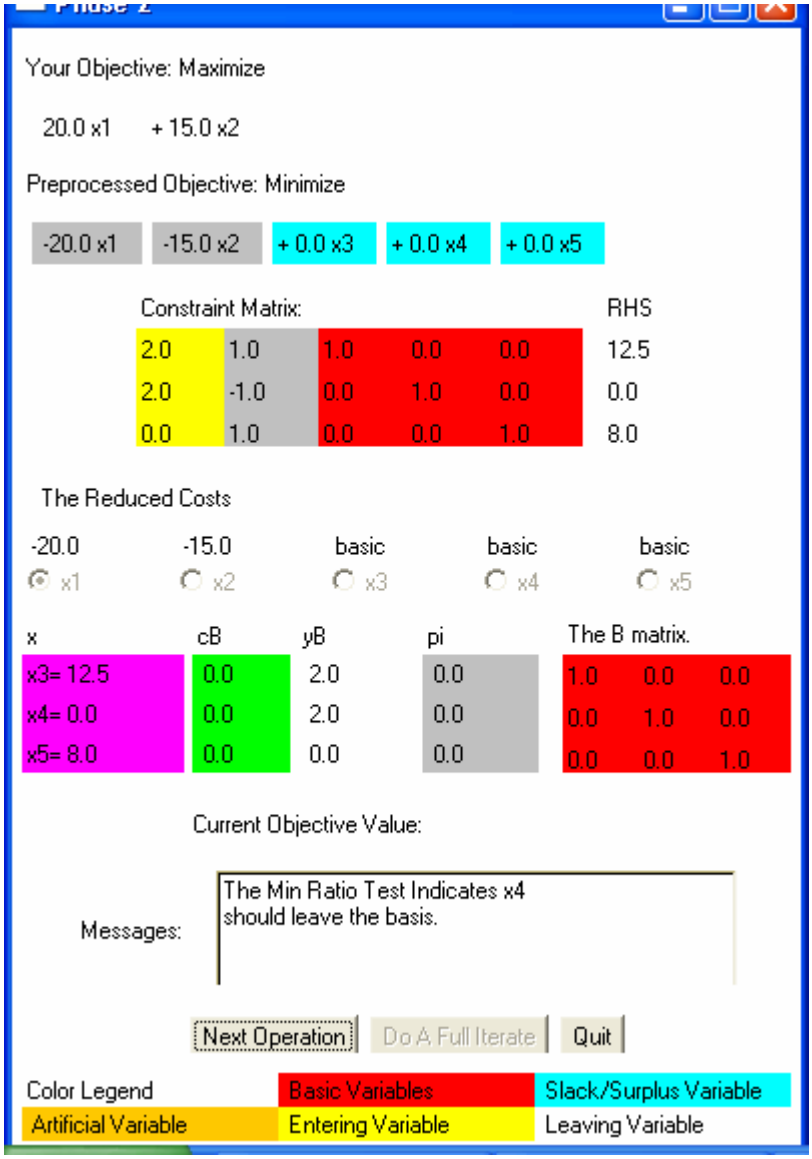

Phase 2
Your Objective: Maximize
20.0 x1 + 15.0 x2
Preprocessed Objective: Minimize
-20.0 x1
-15.0 x2
+ 0.0 x3
+ 0.0 x4
+ 0.0 x5
Constraint Matrix:
RHS
2.0 1.0 1.0 0.0 0.0 12.5
2.0 -1.0 0.0 1.0 0.0 0.0
0.0 1.0 0.0 0.0 1.0 8.0
The Reduced Costs
-20.0 x1
-15.0 x2
basic x3
basic x4
basic x5
x
cB
yB
pi
The B matrix.
x3= 12.5
x4= 0.0
x5= 8.0
0.0
0.0
0.0
0.0
0.0
0.0
1.0 0.0 0.0
0.0 1.0 0.0
0.0 0.0 1.0
Current Objective Value:
Messages:
Calculated reduced costs.
Test for Optimality.
Next Operation
Do A Full Iterate
Quit
Color Legend
Basic Variables
Slack/Surplus Variable
Artificial Variable
Entering Variable
Leaving Variable

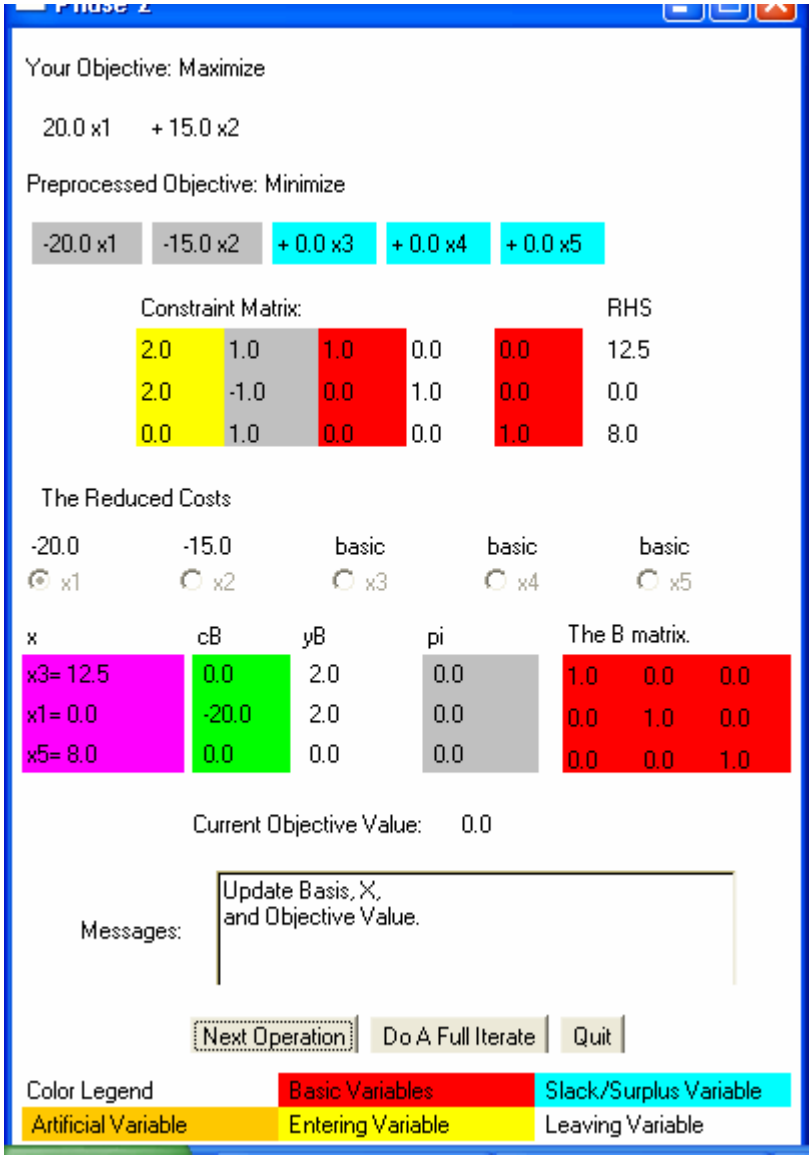

Phase 2
Your Objective: Maximize
20.0 x1 + 15.0 x2
Preprocessed Objective: Minimize
-20.0 x1
-15.0 x2
+ 0.0 x3
+ 0.0 x4
+ 0.0 x5
Constraint Matrix:
RHS
2.0 1.0 1.0 0.0 0.0 12.5
2.0 -1.0 0.0 1.0 0.0 0.0
0.0 1.0 0.0 0.0 1.0 8.0
The Reduced Costs
-20.0 -15.0 basic basic basic
x1 x2 x3 x4 x5
x
cB
yB
pi
The B matrix.
x3= 12.5
x4= 0.0
x5= 8.0
0.0
0.0
0.0
0.0
0.0
0.0
1.0 0.0 0.0
0.0 1.0 0.0
0.0 0.0 1.0
Current Objective Value:
Messages:
Not Optimal!
Pick entering Variable
Next Operation
Do A Full Iterate
Quit
Color Legend
Basic Variables
Slack/Surplus Variable
Artificial Variable
Entering Variable
Leaving Variable


Phase 2
Your Objective: Maximize
20.0 x1 + 15.0 x2
Preprocessed Objective: Minimize
-20.0 x1
-15.0 x2
+ 0.0 x3
+ 0.0 x4
+ 0.0 x5
Constraint Matrix:
RHS
2.0 1.0 1.0 0.0 0.0 12.5
2.0 -1.0 0.0 1.0 0.0 0.0
0.0 1.0 0.0 0.0 1.0 8.0
The Reduced Costs
-20.0 x1
-15.0 x2
basic x3
basic x4
basic x5
x
cB
yB
pi
The B matrix.
x3= 12.5
x4= 0.0
x5= 8.0
0.0
0.0
0.0
0.0
0.0
0.0
1.0 0.0 0.0
0.0 1.0 0.0
0.0 0.0 1.0
Current Objective Value:
Messages:
The entering Variable is x1
Next Operation
Do A Full Iterate
Quit
Color Legend
Basic Variables
Slack/Surplus Variable
Artificial Variable
Entering Variable
Leaving Variable


Phase 2
Your Objective: Maximize
20.0 x1 + 15.0 x2
Preprocessed Objective: Minimize
-20.0 x1
-15.0 x2
+ 0.0 x3
+ 0.0 x4
+ 0.0 x5
Constraint Matrix:
RHS
2.0 1.0 1.0 0.0 0.0 12.5
2.0 -1.0 0.0 1.0 0.0 0.0
0.0 1.0 0.0 0.0 1.0 8.0
The Reduced Costs
-20.0 -15.0 basic basic basic
x1 x2 x3 x4 x5
x
cB
yB
pi
The B matrix.
x3= 12.5 0.0 2.0 0.0 1.0 0.0 0.0
x4= 0.0 0.0 2.0 0.0 0.0 1.0 0.0
x5= 8.0 0.0 0.0 0.0 0.0 0.0 1.0
Current Objective Value:
Messages:
Calculate Search Direction (yB).
Test for Unboundedness
Next Operation
Do A Full Iterate
Quit
Color Legend
Basic Variables
Slack/Surplus Variable
Artificial Variable
Entering Variable
Leaving Variable


Phase 2
Your Objective: Maximize
20.0 x1 + 15.0 x2
Preprocessed Objective: Minimize
-20.0 x1
-15.0 x2
+ 0.0 x3
+ 0.0 x4
+ 0.0 x5
Constraint Matrix:
RHS
2.0 1.0 1.0 0.0 0.0 12.5
2.0 -1.0 0.0 1.0 0.0 0.0
0.0 1.0 0.0 0.0 1.0 8.0
The Reduced Costs
-20.0 -15.0 basic basic basic
x1 x2 x3 x4 x5
x
cB
yB
pi
The B matrix.
x3= 12.5 0.0 2.0 0.0 1.0 0.0 0.0
x4= 0.0 0.0 2.0 0.0 0.0 1.0 0.0
x5= 8.0 0.0 0.0 0.0 0.0 0.0 1.0
Current Objective Value:
Messages:
The Problem is Not Unbounded.
Work continues.
Next Operation
Do A Full Iterate
Quit
Color Legend
Basic Variables
Slack/Surplus Variable
Artificial Variable
Entering Variable
Leaving Variable


Phase 2
Your Objective: Maximize
20.0 x1 + 15.0 x2
Preprocessed Objective: Minimize
-20.0 x1
-15.0 x2
+ 0.0 x3
+ 0.0 x4
+ 0.0 x5
Constraint Matrix:
RHS
2.0 1.0 1.0 0.0 0.0 12.5
2.0 -1.0 0.0 1.0 0.0 0.0
0.0 1.0 0.0 0.0 1.0 8.0
The Reduced Costs
-20.0 -15.0 basic basic basic
x1 x2 x3 x4 x5
x
cB
yB
pi
The B matrix.
x3= 12.5 0.0 2.0 0.0 1.0 0.0 0.0
x4= 0.0 0.0 2.0 0.0 0.0 1.0 0.0
x5= 8.0 0.0 0.0 0.0 0.0 0.0 1.0
Current Objective Value:
Messages:
The Min Ratio Test Indicates x4
should leave the basis.
Next Operation
Do A Full Iterate
Quit
Color Legend
Basic Variables
Slack/Surplus Variable
Artificial Variable
Entering Variable
Leaving Variable


Phase 2
Your Objective: Maximize
20.0 x1 + 15.0 x2
Preprocessed Objective: Minimize
-20.0 x1
-15.0 x2
+ 0.0 x3
+ 0.0 x4
+ 0.0 x5
Constraint Matrix:
RHS
2.0 1.0 1.0 0.0 0.0 12.5
2.0 -1.0 0.0 1.0 0.0 0.0
0.0 1.0 0.0 0.0 1.0 8.0
The Reduced Costs
-20.0 -15.0 basic basic basic
x1 x2 x3 x4 x5
x
cB
yB
pi
The B matrix.
x3= 12.5 0.0 2.0 0.0 1.0 0.0 0.0
x1= 0.0 -20.0 2.0 0.0 0.0 1.0 0.0
x5= 8.0 0.0 0.0 0.0 0.0 0.0 1.0
Current Objective Value: 0.0
Messages:
Update Basis, X,
and Objective Value.
Next Operation
Do A Full Iterate
Quit
Color Legend
Basic Variables
Slack/Surplus Variable
Artificial Variable
Entering Variable
Leaving Variable

По-долу е показан резултатът от последната стъпка на последната операция: