Въпрос №12 ЕМ потенциали. Вълнови у-ния за ЕМ потенциали

$$\begin{cases} \operatorname{rot}\vec H = \frac{\partial \vec D}{\partial t} + \vec J + \vec J_s \\ \operatorname{rot}\vec E = -\frac{\partial \vec B}{\partial t} \\ \operatorname{div}\vec D = \rho \\ \operatorname{div}\vec B = 0 \\ \vec D = \varepsilon\vec E \\ \vec B = \mu\vec H \\ \vec J = \sigma\vec E \end{cases}$$

$\operatorname{div}\vec B = 0$

$\operatorname{div}(\operatorname{rot}()) = 0$

$\vec B = \operatorname{rot}\vec A$ $\vec A$ - векторен магнитен потенциал

$$\operatorname{rot}\vec E = -\frac{\partial \vec B}{\partial t} = -\frac{\partial}{\partial t}(\operatorname{rot}\vec A) = -\operatorname{rot}\frac{\partial \vec A}{\partial t}$$

$$\operatorname{rot}\left(\vec E + \frac{\partial \vec A}{\partial t}\right) = 0 \qquad \operatorname{rot}(\operatorname{grad}()) = 0$$

$$\vec E + \frac{\partial \vec A}{\partial t} = -\operatorname{grad}\varphi$$

$$\vec E = -\frac{\partial \vec A}{\partial t} - \operatorname{grad}\varphi$$ φ - скаларен електрически потенциал

$$\boxed{\begin{aligned} \vec B &= \operatorname{rot}\vec A \\ \vec E &= -\frac{\partial \vec A}{\partial t} - \operatorname{grad}\varphi \end{aligned}}$$ тези равенства дефинират вел. $\vec A$ и φ (не са еднозначно определени)

$$\operatorname{rot}\frac{\vec B}{\mu} = \varepsilon\frac{\partial \vec E}{\partial t} + \sigma\vec E + \vec J_s \iff \operatorname{rot}\vec B = \varepsilon\mu\frac{\partial \vec E}{\partial t} + \mu\sigma\vec E + \mu\vec J_s$$

$\vec B = \operatorname{rot}\vec A$ $\qquad$ $\vec E = -\frac{\partial \vec A}{\partial t} - \operatorname{grad}\varphi$

$$\Rightarrow \operatorname{rot}(\operatorname{rot}\vec A) = +\varepsilon\mu\left(-\frac{\partial^2\vec A}{\partial t^2} - \frac{\partial}{\partial t}\operatorname{grad}\varphi\right) - \mu\sigma\frac{\partial \vec A}{\partial t} - \mu\sigma\operatorname{grad}\varphi + \mu\vec J_s$$

$$\operatorname{rot}(\operatorname{rot}\vec A) = -\varepsilon\mu\frac{\partial^2\vec A}{\partial t^2} - \mu\sigma\frac{\partial \vec A}{\partial t} - \varepsilon\mu\operatorname{grad}\frac{\partial\varphi}{\partial t} - \mu\sigma\operatorname{grad}\varphi + \mu\vec J_s$$

$$\operatorname{rot}(\operatorname{rot}\vec A) = \operatorname{grad}(\operatorname{div}\vec A) - \Delta\vec A \Rightarrow \operatorname{grad}(\operatorname{div}\vec A) - \Delta\vec A = -\varepsilon\mu\frac{\partial^2\vec A}{\partial t^2} - \mu\sigma\frac{\partial \vec A}{\partial t} - \varepsilon\mu\operatorname{grad}\frac{\partial\varphi}{\partial t} - \mu\sigma\operatorname{grad}\varphi + \mu\vec J_s$$

$$\Delta\vec A - \varepsilon\mu\frac{\partial^2\vec A}{\partial t^2} - \mu\sigma\frac{\partial \vec A}{\partial t} = \operatorname{grad}(\operatorname{div}\vec A) + \varepsilon\mu\operatorname{grad}\frac{\partial\varphi}{\partial t} + \mu\sigma\operatorname{grad}\varphi - \mu\vec J_s$$

$$\boxed{\Delta\vec A - \varepsilon\mu\frac{\partial^2\vec A}{\partial t^2} - \mu\sigma\frac{\partial \vec A}{\partial t} = \operatorname{grad}\left(\operatorname{div}\vec A + \varepsilon\mu\frac{\partial\varphi}{\partial t} + \mu\sigma\varphi\right) - \mu\vec J_s} \quad (1)$$

$\operatorname{div}\vec E = \frac{\rho}{\varepsilon}$ (от III^{то} у-ние на М)

$$\operatorname{div}\left(-\frac{\partial \vec A}{\partial t} - \operatorname{grad}\varphi\right) = \frac{\rho}{\varepsilon} \iff -\operatorname{div}\frac{\partial \vec A}{\partial t} - \operatorname{div}(\operatorname{grad}\varphi) = \frac{\rho}{\varepsilon}$$

$\operatorname{div}(\operatorname{grad}\varphi) = \Delta\varphi$ $\qquad$ $-\frac{\partial}{\partial t}(\operatorname{div}\vec A) - \Delta\varphi = \frac{\rho}{\varepsilon}$

$$\boxed{\Delta\varphi + \frac{\partial}{\partial t}(\operatorname{div}\vec A) = -\frac{\rho}{\varepsilon}} \quad (2)$$

$$\boxed{\operatorname{div}\vec A + \varepsilon\mu\frac{\partial\varphi}{\partial t} + \mu\sigma\varphi = 0}$$ - калибровъчно условие на Лоренц

$$\operatorname{div}\vec A = -\varepsilon\mu\frac{\partial\varphi}{\partial t} - \mu\sigma\varphi \Rightarrow$$

$$\Delta\vec A - \varepsilon\mu\frac{\partial^2\vec A}{\partial t^2} - \mu\sigma\frac{\partial \vec A}{\partial t} \ldots \mu\vec J$$

$$\Rightarrow \Delta\varphi - \varepsilon\mu\frac{\partial^2\varphi}{\partial t^2} - \mu\sigma\frac{\partial\varphi}{\partial t} = -\frac{\rho}{\ldots}$$

За непроводяща среда $\sigma = 0$

$$\boxed{\begin{aligned} \Delta\vec A - \varepsilon\mu\frac{\partial^2\vec A}{\partial t^2} &= -\mu\vec J_s \quad (5) \\ \Delta\varphi - \varepsilon\mu\frac{\partial^2\varphi}{\partial t^2} &= -\frac{\rho}{\varepsilon} \quad (6) \end{aligned}}$$

$$\Rightarrow \vec A(\vec r, t) = \frac{\mu}{4\pi}\iiint_{(V)}\frac{J\left(x', y', z', t - \frac{r'}{v}\right)}{\sqrt{(x'-x)^2 + (y'-y)^2 + (z'-z)^2}}\,dV \quad (*)$$ -3-

$\vec r$ - вектора на точката в която търсим реш. ($\vec r = \vec r(x, y, z)$)

$$\Rightarrow \varphi(\vec r, t) = \frac{1}{4\pi\varepsilon}\iiint_{(V)}\frac{\rho\left(x', y', z', t - \frac{r'}{v}\right)}{\sqrt{(x'-x)^2 + (y'-y)^2 + (z'-z)^2}}\,dV \quad (*)$$

x', y', z' - интеграционна точка ($dV = dx'dy'dz'$)

$\vec r'$ - радиуса на инт. точка (разстоянието от т. (x', y', z') до т (x, y, z))

$v = \frac{1}{\sqrt{\varepsilon\mu}}$ - скорост на разпространението на ЕМ вълни

(*) - закъсняващи потенциали